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AN APPLICATION OF INTERFERENCE TO THE STUDY OF THE ORION NEBULA

By H. BUISSON, CH. FABRY, and H. BOURGET

We published in 1911[1] an account of the principle involved in our investigations and of the preliminary attempts that we made to apply interference methods to the study of nebulae.

In 1912 and especially in 1914 we continued these researches and we were able to get a part of the results which we set out to obtain. It should be remembered that the method consists in producing, with the light from the nebula, interference fringes formed at infinity (rings of a thin sheet of air between parallel silvered planes). These rings are projected on a photographic plate which records them; on the other hand, it is arranged so as not to confuse the radiations sent out by different points of the nebula; that is, a clear image of the nebula is obtained on the same plate with the rings. The latter are therefore visible on the surface of the nebula.

DESCRIPTION OF THE DIFFERENT PARTS OF THE APPARATUS

The telescope.—The entire interferential arrangement was mounted on the reflecting telescope of the Observatory of Marseilles, of which the glass mirror, figured by Foucault in 1862, has

[1] Ch. Fabry and H. Buisson, *Astrophysical Journal*, 33, 406, 1911.

a diameter of 80 cm and a focal length of 4.50 m. The telescope is furnished with a driving mechanism worked by electricity, as well as a slow motion in hour angle. To follow the star during the photographic exposure, an objective of 3 m focus and an eyepiece, forming a refracting telescope, have been attached to the tube of the reflector.

The interferometer.—The arrangement shown in section in Fig. 1, for producing and projecting the interference fringes, forms a compact piece which is placed at the open end of the telescope tube,

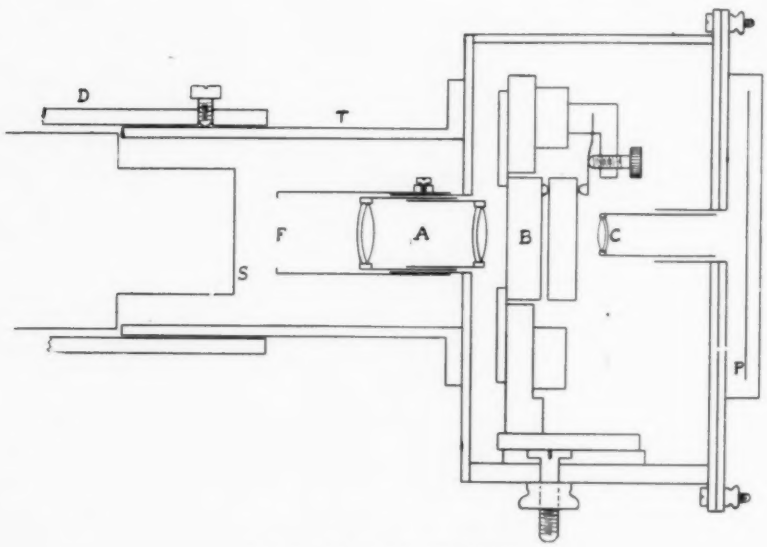


FIG. 1. (About $\frac{2}{3}$ actual size)

centered on the axis of the mirror in such a way that the light reaches it without any additional reflection. This arrangement cuts off a little of the incident light, but only a rectangle 11×14 cm, or 1/30 of the surface of the mirror.

The étalon is composed of two plates of glass 4 cm in diameter and 1 cm thick, the adjacent surfaces of which are silvered and held parallel by three metal blocks of equal thickness. The exact adjustment to parallelism is made by the pressure of three springs, regulated by means of screws. We used thicknesses from 0.1 mm

to 3 mm. For the smallest thicknesses, the separation was produced by three short lengths of wire cut in succession from the same piece. Greater thicknesses were obtained either with small fragments of steel rod, or better, to avoid expansion, with blocks of invar.[1]

The quality of the silvering is of primary importance. We obtained it by cathodic projection, following a method which permitted us to obtain at once exactly and very rapidly the desired thickness of silver. As soon as one plate has been prepared, its transparency and its reflecting power are measured. According to circumstances, we used two sets of silvered surfaces to form the étalon. In the first pair each surface transmitted 0.15 and reflected 0.74 of the incident light; in the other pair, these numbers were 0.30 and 0.60, these values referring to the green radiation of mercury.

After reflection from the mirror of the telescope, the light coming from the nebula forms the image in the focal plane F. It next passes through a pair of lenses A, forming an optical system of short focal length, the focus of which coincides with F. This system is formed by two achromatic lenses of uviol glass in order to avoid the absorption of ordinary flint. Each one of them has a focal length of 86 mm and a diameter of 19 mm and they are 40 mm apart. This whole arrangement is calculated in such a manner that it may have a given focal length (56 mm) and that in the interior of a field with a diameter of 10′ the light reflected from the large mirror shall be completely utilized. The first lens performs the office of field-glass at a distance of only 30 mm from the real image, and diminishes the size of the pencil of rays on the second lens.

The combination consisting of the mirror and the lenses A forms a non-focal system, whose angular magnifying power is 80. In leaving A the light of the nebula seems to proceed from a star at infinity, enlarged angularly 80 times; in addition the bundle of

[1] The blocks were cut by M. Jobin, who gave them a very satisfactory form and who obtained in every case exactly the desired thickness. We shall offer an explanation in the near future of the technique of obtaining the silverings as well as the details of construction of the étalons.

rays passes through the ocular ring of which the diameter is only 1/80 of that of the mirror. It is there that the interference apparatus B is placed, immediately followed by the achromatic objective C, also of uviol glass, having a diameter of 10 mm and 45 mm focal length. In the focal plane of this objective is the photographic plate P. On it we get at the same time the sharp image of the rings (which have not been changed in any way by anything in front of the interference apparatus) and the image of the nebula 80 times larger than if the objective C were pointed directly at the sky.

There also appears on the plate the image of two cross-wires, placed at F in the focal plane of the mirror, to serve as reference lines. The étalon, the lenses A, the objective C, the reticle, and the plate-holder are fitted into a metallic box shown in the cut in Fig. 1. This box is attached to the tube T which fits into the socket D, supported by the frame of the telescope tube. This whole assemblage of parts, which weighs only 4.5 kg, and which takes the place ordinarily occupied by the totally reflecting prism in the Newtonian arrangement, can be taken out and replaced easily.

ADJUSTMENT. COMPARISON RINGS

Certain of the adjustments are made once for all; others should be made anew before each observation. The focusing of the objective C on the photographic plate, and the adjustment of the reticle, in such a way that its image is formed on the plate by the lenses A and C, are accomplished in the laboratory by artificial light.

On the telescope, the setting of the tube T (Fig. 1) is determined so that a clear image of the stars is also shown on the plate. This brings the focus of the mirror into coincidence with that of the system A. This arrangement, like the preceding ones, is independent of the interference apparatus and is made before the latter is put in place.

Before each observation, the parallelism of the silvered surfaces of the étalon is verified, and it is oriented in the box in such a way that the center of the rings coincides as exactly as possible with the crossed threads of the reticle. This arrangement and the

putting into place of the étalon is performed while the apparatus is taken out of the telescope and illuminated by monochromatic light from a mercury-vapor lamp. Before returning the apparatus to place on the telescope, observations are taken of the coincidences among the rings of the various mercury radiations, which give the exact order of interference of each ring.[1]

To measure the interferential photographs of the nebula, it is necessary to have a system of rings, obtained under identical conditions, but with a known monochromatic radiation. This system of rings plays a rôle analogous to the comparison spectrum in ordinary spectroscopes, but, while in the latter case the comparison spectrum and the spectrum to be measured must be made on the same plate, to avoid all displacement, on the other hand the system of comparison rings can be made on a separate plate. The interferential apparatus only must remain unchanged.

A photograph of the comparison rings is made before the exposure on the nebula and another after exposure, leaving the whole apparatus in place on the telescope. At the moment when we wish to make the comparison photographs, we place in the center of the tube of the telescope, about one meter from the opening, a screen of white paper 30 cm in diameter, which is lighted by a mercury lamp held by hand at the opening of the tube. Since the Observatory is supplied only with alternating current, the form of lamp constructed by M. Tian[2] was employed. The lamp is inclosed in a wooden box having a round opening 4 cm in diameter. A glass cell 2.5 mm thick is placed before the opening and contains a weak solution of chromate of potassium intended to absorb the radiations of short wave-length. Since the plates used are practically insensible to green and yellow, only the ray λ 4358 remains to produce the rings. The exposure is about ten seconds.

The necessary exposure for photographing the interference rings of the nebula depends on the radiation employed and the

[1] A. Perot and Ch. Fabry, *Annales de chimie et de physique* (7), **16**, 289, 1899. An explanation of the nature of interference rings by silvered planes will be found in Vol. **15** of the *Records* of the International Bureau of Weights and Measures.

[2] *Journal de physique* (5), **3**, 486, 1913.

thickness of the silverings. Our plates were made with exposures of from one to two hours.

After the preliminary trials, the photographing of interference rings was begun on January 27, 1914, and was continued until March 12; we were able to make 15 plates, some with the radiation H_γ using étalons 1 mm and 2 mm thick, others on the ultra-violet group λ 3728, with thicknesses increasing from 0.13 mm to 2.8 mm.

The accompanying Table I gives a list of all the plates; it gives the date, the radiation used, the thickness and kind of substances separating the silvered surfaces, and finally the exposure-time.

TABLE I

LIST OF PLATES

Date 1914	Radiation Used	Thickness	Blocks	Exposure
January 27	$H_\gamma+H_\beta$	1 mm	Steel	1½ hours
28	$H_\gamma+H_\beta$	2 mm	"	" "
30	H_γ	1 mm	"	1 hour
31	3728	1 mm	"	1½ hours
February 2	"	110 μ	Mica	" "
13	"	130 μ	Wire	" "
14	H_γ	1 mm	Invar	" "
16	3728	1 mm	"	1¾ hours
28	H_γ	2 mm	"	2 "
March 2	3728	0.64 mm	"	" "
3	"	2 mm	"	" "
5	"	2.8 mm	"	" "
7	H_γ	2 mm	"	" "
12	H_γ	1 mm	"	" "

Plate VII reproduces two of the photographs obtained, that of January 31, with the line λ 3728, the other of March 12, with H_γ. They are negatives, that is, the reproductions are identical with the original photographs, save for enlargement.

PRELIMINARY STUDY OF THE SPECTRUM. ABSORBING FILTERS

Before beginning the study of the interference rings, we wished to have a personal and direct knowledge of the spectrum of the nebula and of the relative intensity of the various lines. The publications which we found on this subject lack precision and are sometimes contradictory, which is accounted for by the unequal absorption of the different apparatus used and by the different

PLATE VII

South

H_γ

South

$\lambda 3728$

properties of the photographic plates. We constructed a spectroscope entirely of quartz, very compact and effective as a light-gatherer, the slit of which can be placed at the direct focus of the large mirror. Two 60° quartz prisms, one right-handed and the other left-handed, have square faces 3 cm on a side; the objectives of quartz are 2.5 cm in diameter and 9 cm in focal length. The whole apparatus is attached to a tube identical with that of Fig. 1, fitting into the socket of the telescope. It weighs only 2.7 kg and intercepts but 1/20 of the incident light.

On the Lumière Sigma plates the ultra-violet line λ 3728, not resolved by the spectroscope, is altogether the most intense; the next is the line H_γ; the hydrogen lines of shorter wave-length and the helium lines are much feebler. The group λλ 4861–4959–5006, composed of H_β and two lines of unknown origin, give a perceptible image on the plates.

In our experiments with the interference rings, we have isolated a radiation by absorbing filters as much as possible. The choice of these filters must be made with a great deal of care in order not to weaken the intensity of the line under examination and in order also to diminish as much as possible the intensity of the other lines. We have tried to obtain the most favorable filters by making exact laboratory measurements of the power of transmission. We

TABLE II

λ	Transmission
3728	0.00
4046	0.20
4340	0.40
4861	0.01

employed two groups of filters, one in our study of the line H_γ, the other for the double ultra-violet line. To isolate the line H_γ, the whole ultra-violet end and in particular the line λ 3728 must be weakened; it is necessary also to weaken the green group 4861–5006. The combination employed consists of a Wratten filter of esculin to eliminate the ultra-violet and a Wratten filter *D* (probably a methyl-violet one) which eliminates the green. The transmissive powers of this combination for certain radiations are given in Table II.

To isolate the line λ 3728, the whole visible portion must be eliminated. Violet and blue are eliminated by a screen of nitroso-dimethyl aniline. A fuchsine filter stops the green. These filters are obtained by bathing fixed photographic plates in aqueous solutions. We performed a series of experiments by using solutions of different degrees of concentration, and we chose those which yielded the best results. The transmissive powers are given in Table III.

TABLE III

λ	Transmission
3728	0.50
4046	0.14
4340	0.01
4861	0.00

The filters are put at *S* (Fig. 1) a little in front of the reticle. They are placed at the end of a tube which enters the socket *D* and which permits their interchange without modifying the rest of the installation. The same combination of filters which we used for H_γ was employed to photograph the comparison rings with mercury light.

METHOD OF MEASUREMENT

We are now in possession of three photographs, one from the nebula, and the other two obtained before and after, with the violet line of mercury. Since we know the wave-length of the mercury line, the problem is to determine the wave-length of the radiation which has produced the rings of the nebula. According to the circumstances, this radiation may be a known line like the lines of hydrogen, whose wave-length, however, is modified by the relative motion and will serve to measure the radial velocity, or else it may be a radiation of an unknown element whose wave-length must be determined. The problem of measurement is the same in both cases.

If the nebula has only a motion of translation, the wave-length of a radiation will be the same for all points, the rings are perfectly circular, and the problem is very simple. The diameters of the circles can be measured without reference to the location of their center. Matters are more complex if there are differences of

radial velocity from point to point, because then we can no longer speak of a single value of the wave-length, the rings are deformed, and every point must be defined with respect to the normal to the silvered surfaces (the center of the mercury rings). This set of measurements will give the wave-lengths at the several points and consequently also the distribution of the radial velocities.

Let us examine first the case where there is only a motion of the whole. One of the mercury photographs is placed on a comparator, and the diameters of the successive rings are measured, for example the first five. Let N be the number of the order of the smallest ring measured; it is a whole number which is always known from the observation of coincidences made before the photographic exposures. The problem is to find the order of interference at the center, which can be formulated as $N+\epsilon$. The semidiameters ρ measured from the center obey the law $K\rho^2 = N+\epsilon$.

K is a constant and takes successively the values 0, 1, 2, 3, 4, beginning with the central ring. Combining the five equations which result from the measurement of the five rings, ϵ is calculated together with the constant K. This last quantity can be deduced from the data furnished by the apparatus, but it is more correct to obtain it directly on the plate. The order of interference $N+\epsilon$ is thus determined within a few thousandths of a fringe.

We operate in the same manner on the second mercury plate. We should find precisely the same value for the order of interference if there were no change in the thickness of the étalon or in the index of refraction of the air. In point of fact, the two values obtained differ very little indeed when an étalon with invar blocks has been used. The difference is often less than one-hundredth of a ring and never surpasses 0.03, although the two plates may have been obtained at an interval of two hours and no precaution was taken to eliminate variations of temperature. The average of the two values found is adopted as the value of the order of interference of the violet mercury light.

The plate of the nebula is measured in the same manner. The whole number of the order of interference results from an approximate knowledge of the wave-length of the radiation under observation. The fractional figure is obtained as before by the

measurement of the ring diameters; the corresponding constant K may be deduced from that of the mercury by the observation that it varies in inverse ratio with the wave-lengths. Knowledge of the order of interference in mercury light and of the nebula yields immediately the ratio of the two wave-lengths and consequently that of the radiation of the nebula.

In reality there are differences of radial velocity from point to point and the measurements can only be made point by point. The first thing to be determined in the photograph of the nebula is the foot of the normal to the silvered surfaces whose position on the mercury photographs is defined by the center of the rings. It is in order to correlate the position of this point on our two classes of photographs that we have provided the reticle whose image is reproduced on the photographic plate.

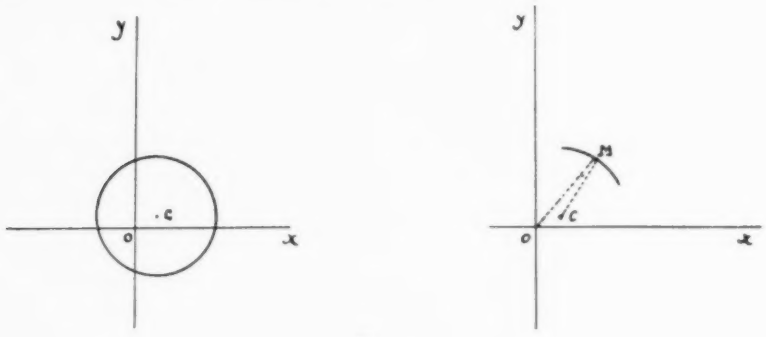


FIG. 2

On the mercury prints the center is determined in the following manner. Let Ox and Oy be the images of the threads of the reticle, and C the center of the rings (Fig. 2). We first orient the print on the comparator in such fashion that the displacement shall be parallel with Ox. With a thread parallel to Oy we then successively set on the two edges of the ring and Oy; from these measurements we deduce the distance from the point C to the axis Oy, that is, the abscissa of the point C with respect to the system of axes xOy. To increase precision, measurements are made on the first five rings. Operating in the same fashion after turning the plate 90°, we obtain the ordinate of the point C.

These co-ordinates, now known, are registered without alteration upon the photograph of the nebula where they determine the foot of the normal to the silvered surfaces.

Passing to the photograph of the nebula we measure, in a known direction from the point O, the distance OM from the origin of the co-ordinates to a point M of one ring. We shall have to deduce the wave-length λ' of the radiation which produced the ring at M. For this purpose we calculate the distance ρ' from the point M to the point C. This is easy because the co-ordinates of these two points are known. The calculation is simplified by the fact that points O and C are very near to each other and hence ρ' differs from OM only by a small amount. Let N' be the known order of interference of the ring which passes through M. $N'+\epsilon'=P'$ is the order of interference which we should have at the point C if the wave-length everywhere had the same value λ' which it had at M. We then have

$$\epsilon'=K'\rho'^2.$$

K' has the same significance as before and is related to K by the formula

$$K'=K\frac{\lambda}{\lambda'}.$$

On the other hand, we know the order of interference $P=N+\epsilon$ which the mercury line λ yields at the center, and we thus have for the desired wave-length the value

$$\lambda'=\lambda\frac{P}{P'}.$$

This measurement can be made point by point on every ring, and from it we deduce the chart of the radial velocities in the nebula, as we shall see.

For these measures we used a comparator constructed by Gaertner of Chicago, after a slight modification of it. On the carriage moved by the screw, a vertical plate-holder is placed, which can turn in its own plane and which carries a divided circle. The reticle of the microscope which serves to make the settings

has but a single wire for the measurement of the mercury rings; whereas for the rings of the nebula, where we must take not a tangent but a definite point, the intersection of two cross-hairs is used.

THE STUDY OF THE RADIAL VELOCITIES

Absolute measurements of radial velocities can be made only by means of radiations capable of emission from terrestrial sources. With radiations of unknown origin nothing but a differential study of the velocities of different points can be accomplished.

For a known radiation we have employed the line H_γ of hydrogen. The measurements were made on photographs that were obtained with an étalon 1 mm thick, that is to say, with rings of the approximate order of 4600. They have been conducted in such a way as to eliminate the wave-length of the mercury line and to compare the radiation of the nebula with that of a hydrogen tube. For this purpose with the same étalon and with the same silvered surfaces, a comparison is made between the mercury line and the artificial line of hydrogen. The whole arrangement represented in Fig. 1 is transported to the laboratory. Here one photograph is taken with the mercury line and one with the line H_γ separated from the other hydrogen lines by a dispersive apparatus. This group of photographs gives the wave-length of H_γ, the value 4338.341 for the mercury line being used. The photograph of the nebula is calculated as we indicated above, the same value for the wave-length of mercury being used. The radial velocity at each point of the nebula results from a comparison between the wave-length at this point and the wave-length which has been found in the laboratory for H_γ. The result is independent of the value adopted for the mercury line, since this has served only as a convenient intermediary, on account of its great intensity.

With rings of the order 4600 a radial velocity of 1 km per second produces a change in the order of interference equal to 0.015, that is, a change of the same order of magnitude as the probable error of a single measurement. To give an idea of the effect of radial velocities on interference rings, let us consider the motion of the earth in its orbit, which produces the maximum radial velocity when the nebula is at 90° from the sun, i.e., in quadrature;

between one quadrature and the next, the difference of radial velocity is 60 km per second. This is equivalent to a change of one whole ring in the interference rings. If it were possible to photograph the rings every day between the two quadratures we should see them contract until every ring took the place of the next preceding one.

In performing the calculations as we have indicated, the absolute radial velocity with respect to the observer is obtained for every point which has been measured on the nebula. An average of these velocities can be taken for a definite surface and can then be corrected for the velocity of the earth in order to obtain the average velocity of the nebula with respect to the sun. On the other hand, the internal movements of the nebula will be represented by calculating the velocities of its several points with respect to this mean velocity.

The differential study of velocities can also be made on photographs obtained with the line of unknown origin λ 3728. This would not permit one to obtain absolute values. This line is finer than H_γ; it therefore permits the employment of fringes of a higher order, upon which the effect of radial velocities is even greater. Since the line is double, we shall choose the thickness of the étalon in such a manner as to have the two systems of rings coincide.

In this kind of study the interference method presents the advantage of yielding the radial velocities of the whole surface of the nebula simultaneously, whereas an ordinary spectroscope permits us to study only those points which are projected on the slit.

Results.—All our prints of the nebula were made with the rings centered upon the region of the trapezium; we utilized the first seven or eight rings. This permits us to study the radial velocity within a circle of about 4′ diameter. The luminous intensity is nevertheless sufficient to permit more extensive measurements, but for this it would be necessary to place the center of the rings in other regions of the nebula, because the measurements lose their precision when rings too remote from the center are utilized.

In the region surrounding the trapezium, the mean radial velocity with respect to the sun is +15.8 km per second; i.e., the

nebula is receding from the sun. This number is the average for values found for 58 points distributed in 12 directions about the trapezium in a radius of about 2′.[1]

Again, the measures show variations of radial velocities from one point to another; this enormous gaseous mass is not at rest relatively. The rings show local deformations in certain regions, indicating, in certain portions of the nebula covering very small areas, irregularities of speed which may amount to about 10 km per second. Movements of this sort are manifested in the region to the southeast of the trapezium in the direction of the star Bond 685. Moreover, there are great collective movements; with respect to the mean velocity, the northeast region is withdrawing at a speed of something like 5 km per second, while the southwest region is approaching at pretty nearly the same speed. In general, the part of the nebula which we have studied has a sort of rotary movement about the line southeast-northwest, but with numerous irregularities.

WAVE-LENGTHS OF THE LINES OF NEBULIUM

We have measured wave-lengths of the ultra-violet group, which with a reflecting telescope is, for photography, the most intense of the whole spectrum. According to Wright[2] this group is composed of two lines, whose wave-lengths he was able to measure on only one plate. The precise determination of the wave-lengths is important because it furnishes a sure basis for attempting the identification of these lines with those of terrestrial elements.

Since it is out of the question to separate the two lines by an absorbing filter, we measure both of them on the same plate,

[1] The velocities found up to the present are:

Keeler (1891)	+17.7
Wright (1901)	+16.2
Vogel (1902)	+17.4
Frost and Adams (1904)	+18.5

The agreement of these numbers with each other and with ours may be considered as satisfactory, especially if one remembers that the velocity is not the same for all points and that the various observations probably do not apply to the same region.

[2] *Astrophysical Journal*, **16**, 53, 1902.

obtained with an étalon of thickness so chosen that the systems of rings of the two radiations are entirely separated. A preliminary measurement made with a difference of path of 250 microns gave a first approximation; the definite measurement was made with a difference of path of 1.3 mm.

To eliminate the effect of radial velocities in the result, the measurement was made according to the method set forth above, by determining the radii of the first five rings in the northwest direction from the trapezium. It is there that the study of radial velocities had shown the fewest inequalities of speed from point to point. The value +17.6 was adopted for the radial velocity of this region.

When the measured lines are compared with the violet radiation of mercury, which is rather far from them in the spectrum, it is necessary to make a small correction to take account of the dispersion of the change of phase by reflection from silver.[1] This correction has been determined by a study in the laboratory. For that purpose we had an étalon of small thickness constructed (130 microns) with the same silvered surfaces, and the resultant rings were measured, radiations of known wave-length being used. The difference in optical thickness in passing from λ 4358 to λ 3728 is only 0.0025 μ; the correction of the wave-length is 0.014 A.

Reduced to the international system the wave-lengths for a source at rest with respect to the observer are:

3726.100
3728.838.

The first of these two lines is the more intense.

The values we have given are exact to a hundreth of an angstrom.

The values given by Wright reduced to the international system are:

3726.25
3728.85 with an uncertainty of $\pm$0.2.

In the list of the lines of known elements, none is found that can be identified with either of these two rays. Before precise measurements had been taken and before we knew that this line

[1] *Ibid.*, **28**, 169, 1908.

was double, the idea had been expressed that it could be attributed to oxygen, which has a rather strong line in this region. Now the wave-length of this line of oxygen in the international system is 3727.35. It falls at about an equal distance from the two lines of the nebula and the interval between both of them and it is much too large for identification to be even contemplated.

ATOMIC WEIGHT OF NEBULIUM. TEMPERATURE OF THE NEBULA

The kinetic theory of gases establishes a correlation between the velocity of agitation of the luminous particles, and thus the width of the lines, and the atomic weight and the temperature of the luminous gas. Now the study of interference rings permits us to obtain the width of the lines by increasing the difference of path and finding to what limit the interference rings are visible. All calculations lead to the following formula: Let T be the temperature of the gas, m the atomic weight of the luminous particles, referred to the ordinary system of atomic weights ($O=16$), and N the order of interference from which point the rings cease to be visible, and we have

$$N=1.22\times 10^6\sqrt{\frac{m}{T}}.$$

Experiment has, in every case studied up to date, verified this formula when for m is substituted the atomic mass of the luminous gas, for the reason that the luminous particles have the same mass as the atom.[1]

The experimental determination of N therefore reveals a relation between m and T. In utilizing the radiations of a known gas, we have a measurement of the temperature of the source. Inversely, if the temperature is known, one can determine the atomic weight of a gas, known to us only by its spectrum. More simply, if the source gives at the same time known and unknown lines, the temperature is eliminated, and the relations of the atomic weights is given by the square of the ratio of the limits of interference.

We looked for the limits of interference for the hydrogen and for the lines of unknown origin, in particular the double ultra-

[1] H. Buisson and Ch. Fabry, *Journal de physique* (5), **2**, 442, 1908.

violet line. Étalons are used of gradually increasing thickness, until the interference rings cease to appear.

Hydrogen.—We used only the ray H_γ, operating by photography. Interference rings are still visible with a difference of path of 4 mm (order of interference 9200). The limit of interference is a little above this number, probably very close to 10,000.

Nebulium.—We studied by photography the double line λλ 3726–3729. To obtain the limit of interference, we worked with increasingly large thicknesses selected in such a manner that the systems of fringes given by the two lines coincide. Exact knowledge of the wave-lengths permits a simple calculation of the differences of path for which these coincidences take place, and we found that it occurred for multiples of 0.5074 mm.

The interference rings still exist for a difference of path of 5.6 mm, that is, for a number of the order of 15,000. The limit is a little higher, and probably approaches 16,500.

This result shows that the unknown gas which emits the double ultra-violet line has an atomic weight higher than that of hydrogen. The ratio of the two atomic weights is $(\frac{16.500}{10.000})^2 = 2.74$. A figure in the neighborhood of 3 is therefore the probable value of the atomic weight of this gas.

A strong green line $\lambda = 5006$ is also due to an unknown gas. We have made, thus far, only visual observations on this radiation, less exact than photographic observations. In spite of the feeble intrinsic brightness of the nebula, rings of which the order of interference reaches 11,000 were distinctly seen. The green line is therefore also emitted by a gas of greater atomic weight than hydrogen. It is not easy to obtain an exact value of the limit, but we consider as probable that this limit is less than 16,500, and, consequently, the green ray is emitted by a gas of lower atomic weight than the body which emits the ultra-violet group.

It is curious to note that the classification of elements recently given by Rydberg leads to the admission, between hydrogen and helium, of two unknown elements having respectively the atomic weights 2 and 3.

Temperature.—The limit of interference of the hydrogen line permits, by means of the formula given above, the calculation of

the temperature of the luminous gas. Assuming 10,000 for this limit, we find a temperature of 15,000 degrees. This number is a maximum; every accessory cause tending to diminish the clearness of the fringes will cause us to find a temperature that is too high, e.g., the differences in radial velocities of gaseous masses radiating to the same point.

CONCLUSIONS

It is to be hoped that further results may be obtained by following the method which we have indicated. There are in particular still to be made the more detailed study of the velocities, on a greater scale, and the study of the green line by photography. The use of absorbents to isolate a radiation is never completely satisfactory. It might be possible to separate the images produced by the various lines by means of dispersive apparatus which would give them all on a single plate. There would be opportunity further to apply the interferential method to other nebulae, in particular to the planetary nebulae.

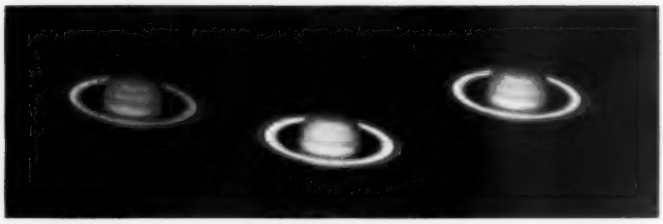
We must emphasize the simplicity of the apparatus used and the ease with which it can be mounted on the telescope. When the silverings have been carefully selected, the interferential apparatus does not cause the loss of much light and permits the study of objects of very feeble intrinsic brightness.

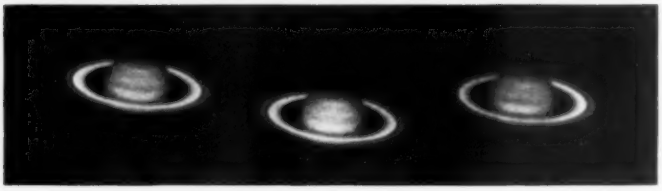
UNIVERSITY OF MARSEILLES

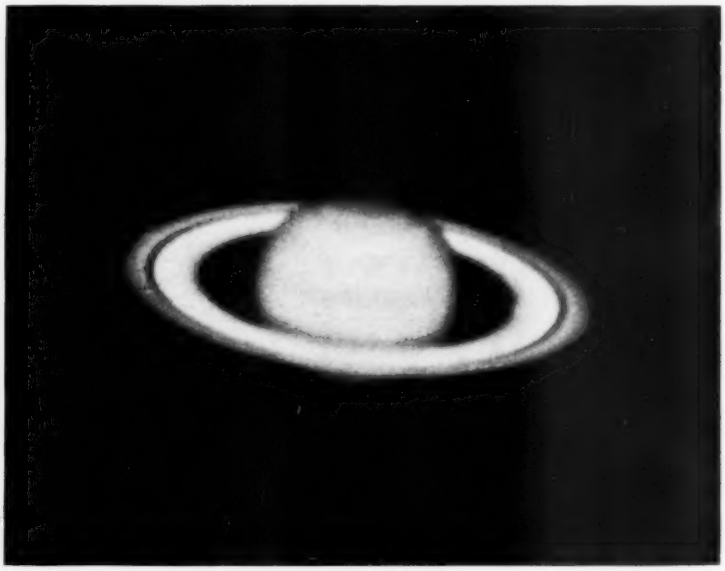
June 1914

PLATE VIII

South







SATURN, 1911, NOVEMBER 19
60-inch reflector, Mount Wilson

E. E. Barnard

PHOTOGRAPHIC MEASURES OF SATURN AND ITS RINGS

By E. E. BARNARD

During part of November 1911, the writer, through the courtesy of Professor Hale, made some observations and photographs of the planet Saturn with the 60-inch reflector of the Solar Observatory at Mount Wilson. See *Year Book No. 11* of the Carnegie Institution for 1912, where a plate of Saturn on 1911, November 19, is given. This plate was also reproduced in the January, 1914, number of *L'Astronomie*. The photographs were made with a Brashear enlarging lens on the 60-inch reflector with the long focus mirror—the exposures being made near the great mirror just outside the tube. To secure sharper contrast a yellow color-filter and a Cramer Instantaneous Iso plate were used. The exposures, with one exception, were ten seconds each.

On the original negative the extreme diameter of the ring of Saturn is 19.01 mm, which is equivalent to the use of a telescope 97.5 meters (320 feet) in focal length.

Twelve images, one of which was purposely overexposed to show the crape ring, were made on the original plate. The crape ring, which extends half-way to the ball, is shown distinctly on the overexposed image (40 seconds' exposure). Several of the other exposures on this plate were injured by bad seeing, but most of them are very perfect and can be measured quite accurately. With the exception of the crape ring they show everything that can be seen with the eye. Indeed, any object as large as one-tenth of a second of arc would readily be visible on this plate—especially if it were an elongated marking. The Cassini division is so clearly shown that the micrometer wires can readily be placed on its edges. It can easily be traced all around the ring. Where it passes in front of the north part of the planet it is very thin but less distinct, and lighter, as if the ball were seen through it. The outer ring where it crosses the ball is lighter than elsewhere.

The relative brightness of the ball and different parts of the ring is well shown. The outer ring and the belts on the globe are about equal in brightness. The outer or brightest part of the inner bright ring is brighter than any portion of the globe. The polar regions are quite dark—much darker than the outer ring. The shadow of the planet on the rings is visible on both sides of the ball, perhaps a little stronger on the following side. The shadow seems to bend out where it reaches the Cassini division, just as it appears in visual observations near opposition.

Herr K. Graff of the Bergedorf (Hamburg) Observatory has recently made a set of measures of the images on the collotype plate as it appeared in *L'Astronomie*. These measures are given in the same publication for March 1914. Graff's results from the paper prints have led me to make a series of measures of the original negative, which, of course, is much superior to the reproduction.

In the work that follows I have measured the original negative and also two glass positives from it, one slightly enlarged and the other very much reduced in size. The original negative and the larger glass positive were measured on a Repsold machine belonging to the Columbia University, for the use of which I am indebted to both Dr. Harold Jacoby and Dr. S. A. Mitchell. The smaller glass positive (a lantern slide) was measured on a Gaertner machine, using a low magnifying power (5 diameters) to produce as sharp an image as possible. The original negative and the larger glass positive (which contained only six of the original images) were measured to see what difference would result from the negative and positive form of image.

As the actual scale of the photographs in seconds of arc is not known, Herr Graff gave his measures in terms of the equatorial diameter of Saturn. For this purpose I have adopted the diameter of the center of the Cassini division, which will give a better representation, as it is very definite and is less liable to halation effect. At the same time it is possible that through some eccentricity of the rings this may not be a constant quantity. It is easy, however, to transfer the measures to the equatorial diameter if any suspicion of change in the Cassini division should occur. The measures are, therefore, independent of scale, and though one cannot give

the quantities accurately in seconds of arc, they are perfectly accurate for the detection of any change that may hereafter occur in the Saturnian ball and ring system. These photographs were made within a few days of opposition and the phase effect would be insensible.

For comparison I give my visual measures of these same quantities, also referred to the Cassini division. A fairly close agreement between the visual and photographic quantities is shown. The most striking difference is in the polar dimensions of the ball—the polar diameter coming out decidedly less in the photographs, thus giving a greater polar compression.

In the original negative the overexposed image shows the crape ring fairly well, though it is faint. To make its edges more definite a greatly underexposed glass positive was made, containing the overexposed image. The measures of the crape ring on this positive are more satisfactory than on the original because of the density of the image of the other rings in the original. The Cassini division does not show, however (though visible on the original). I have, therefore, measured it on several of the other images on the same positive to get the relation of the diameter of the crape ring to that of the Cassini division.

These measures give the ratio of the diameter of the crape ring to that of the Cassini division:

Negative 0.620 Positive 0.627

or, mean ratio 0.623.

The measures of both the negative and the positive make the crape ring extend 0.51 of the space from the inner bright ring to the ball. Visual measures make this ratio a little greater.

By comparing the measures of the same quantities made on both the preceding and following parts of the planet, it was hoped to detect some eccentricity in the rings. There is, however, a very close agreement in these quantities, showing that no measurable eccentricity was visible in any of the elements of the rings at the time of the photographs.

In my measures of the ball and ring system made with the 36-inch refractor of the Lick Observatory in 1894 and 1895, no

eccentricity was apparent in the rings, though special observations were made at the time to detect any displacement of the ball. It would seem, therefore, that if any eccentricity exists in the rings it must be very small.

To show this apparent absence of eccentricity the following results (both for the visual measures with the 36-inch and from the present photographs) are given, where

α is the distance from outer edge of outer ring to center of ball.
β is the distance from center of Cassini division to center of ball.
γ is the distance from inner bright ring to center of ball.

		Visual		From Negative		From Enlarged Positive		From Reduced Positive	
		"	Ratio	mm	Ratio	mm	Ratio	mm	Ratio
α	Prec.	20.102	0.582	9.510	0.571	10.179	0.569	4.953	0.568
	Fol.	20.174	0.584	9.502	0.570	10.214	0.571	4.971	0.570
β	Prec.	17.100	0.495	8.336	0.500	8.952	0.500	4.351	0.499
	Fol.	17.206	0.498	8.326	0.500	8.958	0.500	4.367	0.501
γ	Prec.	12.744	0.369	6.335	0.380	6.778	0.379	3.315	0.380
	Fol.	12.714	0.368	6.343	0.381	6.792	0.379	3.331	0.382

For comparison with my measures, I have referred Graff's measures of the quantities, A, B, D, E, F, G, and (H, I) to his value (C) of the mean diameter of the Cassini division as unity. With the exception of F and G they are identical with my results from the original negative. These last quantities are:

$$F=0.497$$
$$G=0.442$$

These make the equatorial and polar diameters a little larger than my values. The exact agreement of the other values is remarkable.

The measures of the various quantities on the original negative and on the two glass positives follow.

TABLE I

ORIGINAL NEGATIVE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
18.996	16.955	16.650	16.346	12.694	8.323	7.546	0.308	0.301	4.164	4.163	8.326	8.324
19.034	16.988	16.675	16.362	12.706	8.318	7.654	0.302	0.324	4.193	4.164	8.352	8.323
18.984	16.952	16.659	16.366	12.642	8.339	7.601	0.320	0.266	4.159	4.161	8.328	8.331
18.970	16.991	16.666	16.343	12.705	8.316	7.574	0.319	0.329	4.166	4.184	8.324	8.342
19.026	16.962	16.647	16.331	12.677	8.268	7.565	0.301	0.330	4.200	4.179	8.334	8.313
19.034	16.999	16.678	16.358	12.679	8.364	7.608	0.308	0.333	4.170	4.144	8.352	8.326
19.041	16.958	16.658	16.359	12.644	8.342	7.552	0.275	0.324	4.164	4.152	8.335	8.323
(19.200)				(12.412)	(8.567)	7.584						
						(7.692)						
19.012	16.972	16.662	16.352	12.678	8.324	7.586	0.305	0.315	4.174	4.164	8.336	8.326

TABLE II

ENLARGED GLASS POSITIVE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
20.398	18.222	17.902	17.581	13.580	8.992	8.117	0.329	0.312	4.430	4.480	8.926	8.976
20.415	18.239	17.919	17.599	13.584	8.970	8.079	0.328	0.312	4.476	4.473	8.961	8.958
20.447	18.264	17.916	17.568	13.615	8.946	8.146	0.366	0.330	4.471	4.499	8.944	8.972
20.330	18.233	17.878	17.522	13.548	8.927	8.086	0.364	0.347	4.461	4.490	8.974	8.954
20.390	18.197	17.893	17.590	13.520	8.988	8.080	0.314	0.293	4.457	4.448	8.951	8.937
20.380	18.239	17.908	17.578	13.574	8.979	8.072	0.337	0.324	4.464	4.465	8.954	8.954
20.393	18.232	17.903	17.573	13.570	8.967	8.097	0.340	0.320	4.460	4.476	8.952	8.958

TABLE III

REDUCED GLASS POSITIVE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
9.944	8.860	8.719	8.579	6.656	4.285	3.995	0.128	0.153	2.213	2.221	4.355	4.364
9.898	8.827	8.701	8.575	6.631	4.270	3.971	0.138	0.114	2.216	2.215	4.351	4.350
9.931	8.875	8.733	8.591	6.642	4.298	3.961	0.118	0.166	2.212	2.223	4.361	4.372
9.917	8.868	8.714	8.560	6.645	4.282	3.951	0.158	0.150	2.202	2.230	4.343	4.371
9.935	8.877	8.727	8.576	6.651	4.283	3.938	0.147	0.154	2.225	2.219	4.366	4.361
9.925	8.858	8.710	8.563	6.659	4.308	3.958	0.131	0.164	2.190	2.212	4.344	4.366
9.917	8.860	8.724	8.588	6.635	4.295	3.971	0.121	0.151	2.206	2.223	4.354	4.370
9.920	8.856	8.720	8.582	6.668	4.286	3.931	0.143	0.131	2.194	2.240	4.337	4.383
9.930	8.886	8.720	8.554	6.628	4.302	3.981	0.150	0.182	2.198	2.220	4.349	4.371
(9.952)		8.715		(6.520)	(4.392)	3.992			(2.153)	(2.170)	4.349	4.366
						3.938						
						3.979						
9.924	8.863	8.718	8.574	6.646	4.290	3.964	0.137	0.152	2.206	2.223	4.351	4.367

The approximate values of the scale in the above measures, for the three sets, are:

Original negative, 1 mm = 2″.09
Enlarged positive, 1 mm = 1.95
Reduced positive, 1 mm = 4.00

The means of these various measures form Table IV, but the measures of the polar diameter in Table IV have been corrected for the tilt of the planet toward us. In Tables I, II, and III the apparent polar diameter is given.

TABLE IV

	Visual		From Negative		From Enlarged Positive		From Reduced Positive	
	″	Ratio	mm	Ratio	mm	Ratio	mm	Ratio
A.....	40.186	1.164	19.012	1.141	20.393	1.139	9.924	1.138
B.....	35.034	1.015	16.972	1.019	18.232	1.018	8.863	1.017
C.....	34.517	1.000	16.662	1.000	17.903	1.000	8.718	1.000
D.....	34.000	0.985	16.352	0.981	17.573	0.982	8.574	0.983
E.....	25.626	0.742	12.678	0.761	13.570	0.758	6.646	0.762
F.....	17.798	0.516	8.324	0.500	8.967	0.501	4.290	0.492
G.....	16.246	0.471	7.512	0.451	8.006	0.447	3.919	0.450
(*H*, *I*)..	0.517	0.015	0.310	0.019	0.330	0.018	0.145	0.017
N.....	20.528	0.595	10.340	0.620	.....	.....	.....	.....

The only complete measures of the planet and rings made with a large telescope by other observers, with which I am familiar, are those by Professor A. Hall[1] (1884–1887) and by Dr. T. J. J. See[2] (1901), both using the 26-inch Washington refractor, and by Messrs. F. W. Dyson and T. Lewis[3] (1895) with the 28-inch Greenwich refractor. Professor Hall and Professor See did not measure the polar diameter. From their measures I have deduced for comparison (Table V, p. 266) the following ratios with respect to the Cassini division.

It will be noticed that the different sets of visual ratios generally confirm each other, but they differ slightly from the photographic ratios, which are also consistent among themselves.

[1] *Washington Observations*, 1885, Appendix II.

[2] *Astronomische Nachrichten*, **157**, 389, 1902.

[3] *Monthly Notices, R.A.S.*, **56**, 14, 1895.

TABLE V

	Hall		Dyson and Lewis		See	
	"	Ratio	"	Ratio	"	Ratio
A	40.45	1.171	40.590	1.182	39.971	1.171
B	34.95	1.012	34.870	1.015	34.605	1.014
C	34.53	1.000	34.349	1.000	34.138	1.000
D	34.11	0.988	33.828	0.985	33.617	0.986
E	25.75	0.746	25.647	0.747	25.932	0.760
F	17.72	0.513	17.754	0.517	17.240	0.505
G	.....	.....	16.793	0.489	.....	.....
(*H*, *I*)	0.42	0.012	0.521	0.015	0.818	0.024
N	20.52	0.594	20.765	0.604	20.434	0.599

The following letter scheme will explain the quantities given in the tables. The measures in Tables I, II, and III are designated by small letters, and the means of the measures, given in Table IV, by capitals.

a = Outer diameter of outer ring.
b = Inner diameter of outer ring.
c = Diameter of middle of Cassini division.
d = Outer diameter of inner ring.
e = Inner diameter of inner ring.
f = Equatorial diameter.
g = Polar diameter.
h = Width of Cassini division on preceding side.
i = Width of Cassini division on following side.
(*h*, *i*) = Width of Cassini division − mean of *h* and *i*.
j = Middle of Cassini division to preceding limb.
k = Middle of Cassini division to following limb.
l = Middle of Cassini division (pr.) to center of planet.
m = Middle of Cassini division (fol.) to center of planet.
n = Inner diameter of crape ring.

Yerkes Observatory
Williams Bay, Wisconsin
June 1, 1914

NOTE ON THE TRANSPARENCY OF THE OUTER RING

In *Monthly Notices* of the Royal Astronomical Society for June 1914, Mr. Patrick H. Hepburn calls attention to the fact that these same photographs (glass positives of which I had sent to the R.A.S.)

show that the outer ring of Saturn is transparent. Though I had noticed that this ring was brighter where it crossed the ball (see the present paper), I did not attribute the effect to the ball being seen through the rings. It is a fact, however, that the photographs clearly show this ring to be transparent.

In Mr. Hepburn's paper he states that his measures of one of these glass positives show that the inner bright ring is wider on the preceding side than on the following, and he attributes this to some eccentricity in the ring (*M.N.*, **74**, 724). My measures do not show any difference in these quantities, as will be seen from the following, which are the means of the ratios with respect to the Cassini division from the three different plates.

	Ratio
Width of Ring *B* preceding	= 0.111
Width of Ring *B* following	= 0.111

E. E. B.

September 1, 1914

STELLAR WAVE-LENGTH OF λ 4686 AND OTHER LINES IN THE SPECTRUM OF 10 LACERTAE

By EDWIN B. FROST and FRANCES LOWATER

The need has been felt for some years for the accurate determination of the stellar wave-length of the important line which has long been known as the first line of the principal series of hydrogen, but which recent theoretical studies have assigned with some degree of plausibility to helium. The line and its congeners at λ 4542, λ 4200, λ 4026 play an important rôle in the spectra of stars at that end of the spectroscopic sequence which we are accustomed (probably correctly) to call the beginning.

PREVIOUS DETERMINATIONS

The wave-length was originally inferred from objective-prism plates at Harvard as 4688, but later given more precisely as 4685.4. Rydberg derived a wave-length of 4687.88 from his formula,[1] based upon the values he had available for the so-called "sharp" series.

A note on the wave-length of this line in stellar spectra, chiefly in the star 10 Lacertae, with which we are concerned in the present paper, was presented by Frost and Adams at the Washington meeting of the Astronomical Society in December 1902.[2] The value obtained was closely in accord with the value now given in this paper, which is based upon a much greater amount of material.

Meanwhile Professor A. Fowler has been successful in producing the line λ 4686 and others in the laboratory[3] and from accurate measures on plates taken with a concave grating he derives the wave-length 4685.90 (on Rowland's scale, to which are referred all data in this paper). The scale of his spectrogram was 5.5 A per mm, insuring a high degree of accuracy in the result.

[1] *Harvard Annals*, **56**, 103; *ibid.*, **28**, 150; *Astrophysical Journal*, **6**, 236, 1897.

[2] *Publications of the Astronomical and Astrophysical Society of America*, **1**, 207.

[3] *Monthly Notices, R.A.S.*, **73**, 62, 1912.

The line has often been photographed in the flash spectrum at solar eclipses, e.g., by one of us[1] in 1900 at λ 4685.7; and by H. C. Lord at the same eclipse.[2] More accurate wave-lengths were later given by Lockyer, λ 4685.90 (in exact agreement with our stellar wave-length); and by Dyson, 4685.86; and most recently by S. A. Mitchell, λ 4686.00.

CLASSIFICATION OF STAR

This line is sharper in the spectrum of 10 Lacertae ($\alpha = 22^h35^m$; $\delta = +38°32'$; Mag. = 5.0) than in other stars in the collection made with the Bruce spectrograph, as was recognized when the first plates were taken. The other lines of hydrogen and helium are, in this spectrum, also sharper than usual, so that the radial velocity can be pretty well determined from those well known lines. The lines will not bear too high a dispersion, however, and we have found the use of one prism most suitable, as for most of the helium stars. With one prism the scale-value on the spectrogram is 35 A = 1 mm at λ 4686; or 20 A = 1 mm at λ 4089.

The star is placed by Miss Cannon in class Oe5, resembling closely the prototype 30 τ Canis Majoris.[3] It is unfortunate, but hardly avoidable, that so many of the typical stars of this classification have proved to be spectroscopic binaries, for where lines of the second component are visible, the spectrum is likely to be unduly complicated—two spectra instead of one. For instance, the three typical stars of the adjacent classes Oe, Oe5, and B have been found by one of us to be spectroscopic binaries. 10 Lacertae itself, unfortunately enough, proves to vary in its radial velocity, but this does not affect the wave-lengths, as the values from each plate are corrected for the velocity found on that plate from the well known lines of hydrogen and helium. But on a few plates some of the lines could be seen to be double, complicating or frustrating the measurement of radial velocity, so that these plates had to be excluded when wave-lengths were determined.

The general resemblance to 30 τ Canis Majoris as noted at Harvard is confirmed on the Yerkes plates, but the lines λ 4089,

[1] *Astrophysical Journal*, **12**, 344, 1900. [2] *Ibid.*, **13**, 159, 1901.

[3] *Harvard Annals*, **56**, 102.

4097, 4116, 4686 are very much better defined in 10 Lacertae. The resemblance on our plates is much closer in case of 46 ϵ Orionis, the typical star of class Bo, amounting to a practical identity except for the superior sharpness of λ 4686 in 10 Lacertae. We should therefore be disposed to classify 10 Lacertae under group Bo.

The lines above named may be properly designated as sharp with our dispersion. Other conspicuous lines are the helium lines λ 4472 and λ 4026, and Ca K λ 3933, all quite sharp. Calcium H λ 3968 is much sharper than H_ϵ. H_δ is fairly measurable, and has beside it, tolerably distinct, the silicon (?) line λ 4103. H_β and H_γ are measurable with less difficulty than in most B stars.

Of the other helium lines, λ 3964 is fairly clear, λ 4009 is faint on our plates. It is a singular fact that λ 4026.3 of helium seems to be single and not confused by the line of the "sharp" series at λ 4026, which Pickering designated as ϵ'. λ 4024 of helium was not measured on any of the plates, but seems to be faintly present.

On plates 89, 419, 431, 439, 3481, several lines were double or complex, showing the presence of the fainter component; the principal ones were H_β, H_γ, H_δ, and 4388.

MEASUREMENTS AND RESULTS

All the plates were measured by each of us, and from the well known hydrogen and helium lines a first approximation to the radial velocity on the plates was made. Titanium and iron furnished the comparison spectrum. The wave-lengths, to a first approximation, of lines required were then derived after applying the proper correction for the radial velocity, the results for each observer being left separate. A second approximation for velocity and wave-lengths was then made, of course differing only slightly from the first. The results from the two observers were combined, and then, utilizing these wave-lengths of the lines λ 4089, 4097, 4116, and 4686, a final value of the radial velocity for each observer was obtained. These wave-lengths are given in Table I.

It will not be necessary to tabulate the separate determinations of wave-length for the four lines, which were in excellent accord among themselves and between the two observers. The condensed results are as tabulated.

The last three lines are of considerable importance in stellar spectra of classes Oe to B2. Approximate wave-lengths were given by Miss Cannon from objective-prism plates, and the chemical

TABLE I

Frost :4685.897±0.016 (15 plates) Lowater: .903±0.018 (15 plates)	4116.338±0.015 (12 plates) .328±0.018 (14 plates)
Means: 4685.90	4116.33
Frost :4097.536±0.011 (12 plates) Lowater: .564±0.014 (14 plates)	4089.127±0.012 (13 plates) .115±0.011 (14 plates)
Means: 4097.55	4089.12

origin and position of these lines have been studied in the laboratory by others, most recently by Lunt, by Lockyer, Baxandall, and Butler, and by Sir William Crookes. The values obtained in the spark spectrum of silicon are:

Lunt[1]	4089.00	.....	4116.35	
Lockyer, Baxandall, Butler[2]	.04	4097.49	.29	
Crookes[3]	.02	.....	.....	

Crookes does not find λ 4116 in the spectrum of pure elementary silicon, and he gets the wave-length 4097.02 for a silicon line. This cannot be the stellar line above measured by us at λ 4097.55, which has been attributed by Lunt to some impurity in his silicon, and by Lockyer and his collaborators to nitrogen. For this, laboratory values have been published as follows:

Neovius	4097.4
Hemsalech	4097.3
Exner and Haschek	4097.43

From plates of ε Orionis taken with an objective-prism, Lockyer and his colleagues find for these lines wave-lengths as follows:

4089.14 4097.59 4116.54

The other lines of the so-called "sharp" series of hydrogen, at λ 4542 and 4200, were vague and broad on these spectrograms, so

[1] *Annals of the Cape Observatory*, **10**, Part 2, 1906.

[2] *Proc. Roy. Soc.*, **82** A, 532, 1909. [3] *Ibid.*, **90**, 512, 1914.

that their wave-lengths could be determined only roughly. We are not familiar with any stellar spectrum in which they are well defined. The results are:

Frost	λ 4541.8	(10 plates)	4200.3	(6 plates)
Lowater	4542.0	(10 ")	4200.3	(5 ")
Mean	4541.9		4200.3	

We have mentioned above that the next line of this series, at about λ 4026, is not measurable, either because of its faintness or because it is masked by the excellent helium line at λ 4026.3. The

TABLE II

RADIAL VELOCITY OF 10 LACERTAE

PLATE	DATE	G.M.T	FROST		LOWATER		MEAN KM	QUALITY	TAKEN BY
			No. of Lines	Km	No. of Lines	Km			
89	1903, Sept. 25	14^h44^m	.....	.....	.....	.....	.....	.....	F
128	Oct. 17	18 34	8	−11.6	9	−10.1	−10.9	g.	F
150	Oct. 24	14 50	9	13.6	8	16.3	14.9	v. g.	A
409	1904, Nov. 1	15 14	11	14.7	10	15.5	15.1	v. g.	B
419	Nov. 11	14 31	9	9.6	9	10.5	10.0	g.	F
431	Nov. 15	14 37	10	9.9	10	7.8	8.9	g.	B
439	Nov. 18	14 34	11	12.0	8	8.9	10.4	v. g.	F
556	1905, July 14	19 07	5	5.4	4	4.6	5.0	f.	B
852	1906, Sept. 17	15 45	11	15.6	10	19.9	17.8	g.	F
1080	1907, June 10	20 46	11	18.4	10	15.6	17.0	v. g.	F
1118	July 26	19 38	10	19.2	8	16.0	17.7	v. g.	F
1239	Nov. 22	14 21	9	10.5	9	8.7	9.6	v. g.	B
1662	1908, July 31	20 49	10	15.8	9	12.5	14.2	v. g.	B
2165	1909, Oct. 29	15 10	9	15.8	8	18.4	17.1	g.	L B
3473	1913, Aug. 29	20 09	7	8.5	7	6.9	7.7	p.	F
3481	Sept. 4	19 17	8	10.8	9	11.5	11.2	g.	F
3483	Sept. 5	15 47	6	9.2	8	9.0	9.1	g.	L
3490	Sept. 8	18 22	9	8.6	8	9.4	9.0	g.	B
3496	Sept. 10	19 55	10	14.2	10	12.2	13.2	g.	L

In the last column A = Adams, B = Barrett, F = Frost, L = Lee. Mr. Sullivan assisted as usual. g. = good; f. = fair; p. = poor; v. = very.

next line of this series beyond this, at λ 3924, is too far from the center of good focus for measurement, and it was not measured on any plate; but on strongly exposed plates a line is seen near this point.

For plate 89 the results must be given separately for the two components (Table III).

TABLE III

	Single Lines	Double Lines	
Frost.....	5 −14 km	3 −57 km	+22 km
Lowater.....	6 −18	3 −42	+26
Mean.....	−16 km	−50 km	+24 km

The above data are insufficient for indicating the period of this star, but there seems to be little to suggest a period of less than several days. The range on the best of the above plates is about 8 km, extreme range for the different plates, 13 km. The motion of the system is probably about −12 km, almost wholly due to the solar motion.

It might be thought necessary to correct the wave-lengths given above for the discrepancy of about +5 km found to be inherent at present in our determinations of radial velocity of B stars, but a moment's reflection will show that these wave-lengths depend upon the laboratory values of the hydrogen and helium lines, and they should therefore be unaffected by this systematic error.

YERKES OBSERVATORY
September 3, 1914

THE ÅNGSTRÖM COMPENSATION-PYRHELIOMETER AND THE PYRHELIOMETRIC SCALE[1]

By A. K. ÅNGSTRÖM

I propose to discuss here a small source of error which has appeared in the construction of the Ångström compensation-pyrheliometer. The correction involved is shown not to exceed 2 per cent, but since the instrument has been accepted as a standard instrument by the International Congress at Oxford, and since this correction is of particular interest in the construction of instruments of this type, a somewhat detailed discussion may be desirable.

In this paper, therefore, I intend (1) to deduce the magnitude of the correction theoretically, and (2) to give an experimental determination of this correction.

As is well known, the principal part of the Ångström pyrheliometer consists of two equally blackened manganin strips, to the middle points of which are connected the two junctions, m and n, respectively, of a thermo-element. The two metal strips are soldered at both ends to the metal supports O and P, O' and P'.

The measurement of the radiation is made as follows: One of the strips is exposed to the radiation, the other screened from it. We then measure the current which must flow through the screened strip in order to keep the attached junction of the thermo-element in the middle of the strip at the same temperature as the junction at the middle of the illuminated strip. The two strips are then interchanged so that the strip previously covered is illuminated, and vice versa. The intensity of the radiation is computed from the mean value obtained from three such measurements of the current.

If the distribution of temperature in the two strips is exactly the same, this procedure will naturally give entirely correct results. But this will not be the case if the distribution of temperature is not the same. Such a difference in distribution is due principally to two causes.

[1] This paper will appear also in the *Meteorologische Zeitschrift*.

1. The illuminated strip is warmed from the surface, whereas the shaded strip is heated throughout its entire volume by the electric current. The consequence is that the temperature-gradient, taken from the surface inward, is not quite the same in the two cases. It is easy to show that in practice (length of the manganin strips 2 cm, thickness 20 μ, thickness of the coating of soot 10 μ) the error resulting from this cannot exceed one-half of 1 per cent.[1] This is on the assumption that the conductivity of

[1] We consider first the problem of ascertaining the illumination necessary to impart a temperature t_2 to the back surface of a thin strip consisting of two portions laid face to face. We obtain easily

$$I = t_2(\alpha+\beta+\alpha\beta\mu)$$

where

$$\mu = \frac{c_2 d_1 + c_1 d_2}{c_1 c_2}$$

and α is the rate of efflux of heat per element of area of the illuminated face and β is the rate of efflux of heat per element of area of the back surface. d_1 and d_2 are the thicknesses, c_1 and c_2 the thermal conductivities, of the two substances 1 and 2.

In the case where the second portion is heated by an electric current, producing a quantity of heat W in an element of volume determined by the element of area and the thickness of the strip, we can deduce

$$W = t_2 \frac{\alpha+\beta+\alpha\beta\mu}{1+\alpha\mu-(\alpha d_2/2c_2)},$$

whence

$$\left(\frac{I}{W} = 1+\alpha\mu-\frac{\alpha d_2}{2c_2}\right)$$

If now, as in the compensation-pyrheliometer, the first substance is soot, the second manganin, c_1 may be neglected relatively to c_2, and we get

$$\left(\frac{I}{W} = 1+\frac{\alpha d_1}{c_1}\right)$$

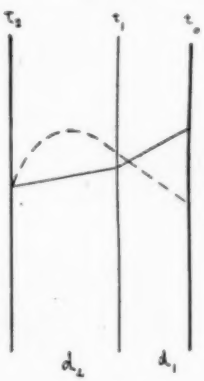


FIG. 1

If then we set: $\alpha = 0.00026$ (MacFarlane); $d_1 = 0.001$; $d_2 = 0.002$, and $c_1 = 0.57 \times 10^{-4}$ (conductivity of air), we obtain

$$\left(\frac{I}{W} = 1.0046\right)$$

wherein we have taken the conductivity of soot as equal to that of air, which probably gives much too small a value for c_1, and hence too great a value for $\alpha d_1/c_1$. We therefore conclude that the correction to W required by (1) alone cannot exceed one-half of 1 per cent.

the soot is not less than that of the air. Such a result can also be deduced from the empirical investigations of Kurlbaum.

2. A second cause of the difference in temperature-distribution between the strips arises if the exposed strip is illuminated along a part of its length only. This in fact is the case. In a pyrheliometer (No. 158) used by me in expeditions to Algiers and to Mount Whitney, it was found that about one millimeter at the edges of the illuminated strip (the length of which was 2 cm) was in the shadow of the diaphragm.

An idea of the magnitude of this border-effect, which is an important source of error in all instruments of similar construction, will be gained from what follows. I will assume the following simplifications: that the temperature is the same throughout each cross-section, and that the temperature of the supports O, P, and O', P' is the same as that of the surroundings. If, further, we assume that the rate at which each surface-element loses heat is proportional to its difference of temperature from the surroundings $(-kt)$, we obtain for the determination of the thermal condition of the electrically heated strip

$$W a dx = k t a dx - abc \frac{d^2t}{dx^2} dx \qquad (1)$$

where W is the heat evolved per element of area of the strip, a the breadth of the strip, b its density, and c its internal thermal conductivity.

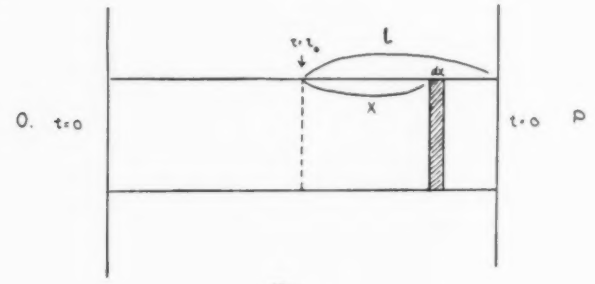


FIG. 2

From (1) we obtain

$$\frac{d^2t}{dx^2} - \frac{kt}{bc} + \frac{I}{bc} = 0 \qquad (2)$$

The general solution is of the form:

$$t = Ae^{-\mu x} + Be^{\mu x} + S$$

where

$$\mu = \sqrt{k/bc} \qquad S = W/k$$

If the constants A and B are determined from the conditions $t = 0$ when $x = l$ and $dt/dx = 0$ when $x = 0$ (the latter follows from symmetry), we obtain

$$A = B = -\frac{S}{e^{-\mu l} + e^{\mu l}}$$

For the temperature t_0 at $x = 0$, i.e., the temperature at the junction, we have

$$t_0 = S\left[1 - \frac{2}{e^{-\mu l} + e^{\mu l}}\right] \tag{3}$$

This temperature t_0 must be equal to the temperature of the middle point of the illuminated strip. For the illuminated part of this strip there holds, in analogy with (1), the equation:

$$t = A_1(e^{-\mu x} + e^{\mu x}) + S_1 \tag{4}$$

where

$$S_1 = I/k,$$

I being the intensity of radiation per element of area. For the part not illuminated we have

$$\frac{d^2t'}{dx_1^2} - \frac{k}{bc}t' = 0 \tag{5}$$

which gives the solution:

$$t' = Me^{-\mu x'} + Ne^{\mu x'}$$

where A_1, M, and N are determined by the conditions that at $x = d$, $t_1 = 0$, and at $x = 0$, $t_1 = t$, and the third condition,

$$\left(\frac{dt}{dx}\right)_{x=l-d} = \left(\frac{dt'}{dx_1}\right)_{x_1=0}$$

whence we obtain

$$\left.\begin{aligned} Me^{-\mu d} + Ne^{\mu d} &= 0 \\ -M + N &= A_1[e^{\mu(l-d)} - e^{-\mu(l-d)}] \\ M + N &= A_1[e^{\mu(l-d)} + e^{-\mu(l-d)}] + S_1 \end{aligned}\right\} \tag{6}$$

and from (6)

$$A_1 = \frac{S_1}{2}\left[\frac{e^{-\mu d}+e^{\mu d}}{e^{-\mu l}+e^{\mu l}}\right],$$

which together with (4) gives

$$t_0 = S_1\left[1-\frac{e^{-\mu d}+e^{\mu d}}{e^{-\mu l}+e^{\mu l}}\right]. \tag{7}$$

From (3) and (7) follows the relation desired:

$$\frac{I}{W} = \frac{\psi(l)-1}{\psi(l)-\psi(d)}, \tag{8}$$

where

$$\psi(l) = \frac{e^{-\mu l}+e^{\mu l}}{2}.$$

Knowing l, d, and μ, we can now determine from (8) the relation between the quantity of heat generated by the current per element of area, and that imparted by the radiation. We thus obtain a value $(1+\delta)$, where δ is the correction which must be made upon the value for the quantity of heat computed from the strength of the current. If c is set $=0.52$ (from Jaeger and Diesselhorst), and k is set $=0.00018+0.00026=0.00044$ (from MacFarlane), the computed value for μ is 0.65, since the thickness of the strip is $20\ \mu$.

The length of the strip is 2 cm; hence $l=1$ cm. Hence we have

$$\frac{I}{W} = \frac{0.2188}{1.2188-\psi(d)}$$

The following table is instructive:

d	I/W
0.05	1.0025
0.1	1.011
0.15	1.023

The correction increases rapidly as d increases. In our special case d was estimated at about 1 mm. The corresponding correction, according to the theoretical considerations, is 1.1 per cent.

Experimentally the correction was determined by replacing the diaphragm belonging to the instrument by another, which permitted the illumination of the whole strip. In both cases the

readings were compared with those of a pyrheliometer belonging to the Smithsonian Institution (A.P.O. 9). The mean value of five series gave in the first case the relation

$$\frac{\text{A.P.O. 9}}{\text{Ångström}^1} = 1.0430$$

and in the second case

$$= 1.0299$$

The difference of 1.3 per cent is, therefore, to be ascribed to a border-effect, the sign of which is such that the readings give values too small. In my opinion, therefore, all measurements made hitherto with the Ångström pyrheliometer should be corrected by +1.3 per cent. As I have since found that my pyrheliometer No. 158 gives values about 1 per cent lower than most Ångström pyrheliometers, the difference between the International scale and that of the Smithsonian Institution becomes after this correction about 2.5 per cent (±1 per cent).[1]

As regards this difference, it is perhaps too early to make a definite statement. It must be conceded, however, that Abbot's method (the water-flow pyrheliometer) has a distinct tendency to give values too high. Thus in 1908 the difference between the two scales was 9.2 per cent,[2] in 1911 it was 5.5 per cent,[3] and in 1913 it was 3.9 per cent,[4] whence it is evident that the Abbot scale after revised measurements has gone considerably downward. On the other side, all the sources of error in the Ångström method tend to result in too small values, and the possibility of a necessary extra per cent of correction is not excluded. It is, therefore, probable that the true pyrheliometric scale lies between limits which do not differ by more than 2.5 per cent of the quantity of heat measured.

In this connection it may be of interest to mention briefly an experimental result which my friend Professor C. G. Abbot has

[1] Vol. 3 of the *Annals* of the Astrophysical Observatory of the Smithsonian Institution gives the difference as 3.9 per cent.

[2] *Ibid.*, **2**, 47.

[3] Abbot and Aldrich, *Astrophysical Journal*, **33**, 125, 1911.

[4] *Annals*, **3**, 72.

kindly placed at my disposal. It follows from this that the measurements with the compensation-pyrheliometer are practically independent of pressure. Abbot inclosed the Ångström pyrheliometer in a glass tube which could be evacuated, and compared the readings at different pressures with those of a pyrheliometer outside. He found that at very low pressures (0.1 mm) the instruments gave values differing by 4.9 per cent from those obtained at atmospheric pressure. At a pressure of 34 cm nothing remained of this effect. When the ratio of the readings of the two pyrheliometers is set $=1$ at atmospheric pressure, the value of the ratio at 34 cm comes out 1.0001 (eight measurements).

The erroneous result at very low pressures is probably to be ascribed to a magnification of the above-mentioned border-effect.

SUMMARY

Theory and experiment show that the readings of the Ångström pyrheliometer must be corrected by about $+1.3$ per cent. The difference between this scale and that employed by the Smithsonian Institution appears, therefore, to be 2.6 per cent.

Experiments show that the constant of the instrument is independent of pressure within the limits of practical work.

I am much indebted to Professor C. G. Abbot for valuable discussions with him.

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UPSALA
May 1914

[1] *Ibid.*

A SHORT METHOD FOR DETERMINING THE ORBIT OF A SPECTROSCOPIC BINARY

By HENRY NORRIS RUSSELL

So many ways of determining the orbit of a spectroscopic binary have been published that an additional one would appear to demand some justification. If the problem were only to determine the orbital elements from a given velocity-curve, of the exact theoretical form consistent with elliptic motion, any one of several existing methods (some of them of great mathematical elegance) would afford a satisfactory solution. But the computer has actually to fit such a curve to a series of points, derived from observations which are often affected by considerable errors; and this makes the problem much more difficult, so that it is not unusual to read that several trials were necessary to reach preliminary elements which appeared near enough to the truth to be used as a basis for a correction by least squares, while sometimes a second approximation by this laborious method has been necessary.

For nearly circular orbits, the analytical method of Wilsing, as extended by the writer,[1] affords a satisfactory solution; but it is convenient only for eccentricities less than 0.3. King's graphical method[2] also makes it possible to use the whole course of the velocity-curve in finding the preliminary elements, and is applicable to orbits of any eccentricity. Both these methods depend upon the velocities at equal intervals of time, as read from a freehand curve drawn to represent the observations. The method here proposed is free from this limitation, and enables the computer to fit his elements directly to the observations themselves, though it may be applied also to a freehand curve. It is equally applicable to orbits of all eccentricities.

Let it be supposed that a period, satisfactory at least as a first approximation, has been found, and that the observations have been assembled on this period, and combined into normal places,

[1] *Astrophysical Journal*, **15**, 252, 1902.

[2] *Ibid.*, **27**, 125, 1908.

in the usual manner. We know for each normal place the radial velocity ρ and the time t referred to some arbitrary initial epoch. If M is the mean anomaly in the orbit, T the time of periastron passage, and n the mean motion, we have

$$M=n(t-T).$$

Setting $M_0=nT$, we may write

$$nt=M+M_0 \tag{1}$$

so that the value of $M+M_0$ is known for each observation.

The fundamental equation for the radial velocity may be written, with the usual notation,

$$\rho=\gamma+Ke\cos\omega+K\cos(v+\omega)=G+K\cos(v+\omega) \tag{2}$$

where v is the true anomaly in the orbit. The maximum velocity is $G+K$, and the minimum $G-K$, so that G and K may be estimated at once from a freehand curve. We may then write (2) in the form:

$$\cos(v+\omega)=\frac{\rho-G}{K} \tag{3}$$

and compute $v+\omega$ from each observed value of ρ. If we subtract the corresponding values of $M+M_0$ from each of these, we shall have values of $(v-M)+(\omega-M_0)$. The second part of this expression is constant, while the first is the equation of the center in the elliptic motion. During a revolution this varies between equal positive and negative limits which depend only on the eccentricity, and are nearly proportional to it, as is shown in the following table.

TABLE I

Eccentricity	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Max. equation of center	11°.5	23°.0	34°.8	46°.8	59°.2	72°.3	86°.4	102°.3	122°.2

If the values of $v-M+\omega-M_0$ are plotted against those of $M+M_0$, we obtain a diagram which, since it represents the relations between the mean and the true anomalies, we may call the anomaly diagram. If on this diagram a curve is drawn to represent

the plotted points, half the difference between its maximum and minimum ordinates will be the greatest value of the equation of the center, from which e may be found at once by means of Table I.[1] The mean of the maximum and minimum ordinates will be the value of $\omega - M_0$. The instants when $v - M + \omega - M_0$ has this value are those of periastron and apastron passage, the former corresponding to the ascending branch of the curve, which is always the steeper. The abscissae of the corresponding points of the curve are M_0 and $M_0 + 180°$. The values of e, M_0, and ω are now known, and the remaining elements may be found at once from K and G.

The principal advantage of this method is that the form of the curves which give $v - M$ as a function of M depends upon e alone. For orbits of the same eccentricity, a change in M_0 (that is, in T) shifts the curve on the anomaly diagram horizontally, and one in $\omega - M_0$ shifts it vertically, without otherwise altering it. It is therefore possible to draw upon tracing-cloth, once for all, a set of curves with $v - M$ as ordinates and M as abscissae, for every 0.05 or 0.10 of e. Such a diagram may very easily be prepared with the aid of the tables for the true anomaly in elliptic orbits given in Vol. 2 of the *Publications of the Allegheny Observatory* (pp. 158–190). By superposing this sheet upon the anomaly diagram for any star (plotted to the same scale), and shifting it in both co-ordinates (keeping the co-ordinate axes on the two sheets parallel), it is easy to determine what value of e, and what position of the periastron in time and space, will give the best representation of the observations.

In this connection it must be remembered that the values of $v + \omega$ determined from values of its cosine near ± 1—that is, from radial velocities near the maximum or minimum—are subject to considerable uncertainty. The corresponding points should be distinguished from the others in the plot, and attention directed mainly to obtaining a good fit for the well-determined points. It

[1] If ϵ is the maximum equation of the center, in degrees, the approximate formula

$$e = \tfrac{7}{8}\left\{\frac{\epsilon}{100} - \frac{1}{10}\left(\frac{\epsilon}{100}\right)^3\right\}$$

gives results which are correct within 0.001 for values of e less than 0.80, and only 0.005 too great when $e = 0.90$.

is best to compute $v+\omega$ directly from the observed velocity, except when this is near the maximum or minimum, in which case the reading of a smooth curve drawn to represent the course of the observations is usually preferable. It will usually be found that e cannot be varied by more than 0.02 on either side of the best value, or $\omega-M_0$ by more than a degree or two, without introducing inadmissible discordances with the observations. The precision with which M_0 is defined depends upon the eccentricity. When this is large, it may be uncertain by less than a degree; when it is small, by many degrees.

When a satisfactory fit has been obtained, the theoretical velocity-curve, corresponding to the assumed elements, may be computed at once. Keeping the tracing-cloth fixed, relative to the anomaly diagram below it, the values of $v-M+\omega-M_0$ corresponding to any desired values of $M+M_0$ may be read off, by interpolation between the curves for the adjacent values of e. Adding, we have the values of $v+\omega$, and the velocity is given by (2). Near periastron, in very eccentric orbits, the curves run so steeply that the values of $v-M+\omega-M_0$ cannot be read from them with precision, and it is well to use the Allegheny tables to find v.

If the points on the anomaly diagram cannot be satisfactorily represented by means of a curve of the theoretical form, it is probable that the assumed values of K and G are in error. Even in such a case, good approximations to e, ω, and M_0 may usually be obtained from the diagram. If a velocity-curve is computed from these values, and the assumed K and G, and plotted along with the observations, it is easy to see what changes in the maximum and minimum; and hence in K and G, are needed. It is still better to plot the residuals for the individual observations from this first approximation against the computed velocities, and draw a straight line to represent them as well as may be. The readings of this line opposite the originally assumed maximum and minimum velocities give the corrections which must be applied to them in order to get the best representation of the observations with the values of e, ω, and M_0 derived in the first solution. From these, new values of K and G may be found, and a second anomaly diagram prepared, from which new values of e, ω, and M_0 are found as

above. Such a second approximation is necessary only in difficult cases, and a third is seldom if ever requisite.

The method here developed is closely analogous in principle to the graphical method of Dr. King, but has the advantage that a single diagram takes the place of the numerous "protractors" which must be constructed for each separate value of the eccentricity. It is also probable that the graphical processes, which demand the exact drawing of lines and measurement of angles will consume more time than the methods here suggested. After the observations are plotted, and the values of K and G estimated (which is necessary in both methods), the individual values of $v+\omega$ can be read from a slide-rule as fast as they can be written down. It takes then but a few minutes to plot the anomaly diagram, and a few more to adjust the tracing-cloth over it, except in cases of unusual difficulty. The preparation of this latter diagram, which is done once for all, with the aid of the Allegheny tables, takes two or three hours. The complete solution for an ordinary star, from the beginning of plotting the observations to the determining of the elements and the residuals from the theoretical curve, may be made in less than an hour.

As an example of the practical application of this method the discussion of α Draconis may be given in detail. An orbit of this system, determined by Harper[1] from 59 plates taken at the Dominion Observatory, Ottawa, after a thorough discussion by least squares, may be taken as a standard of comparison.

The period, 51.38 days, was found by comparison with observations made several years earlier, and no correction to it was attempted in the final adjustment. Harper's normal places, with phases counted from a preliminary determination of the periastron, are given in the first two columns of Table II, and the weights assigned by him follow. The remaining portion of the table gives the solution by the method of this paper.

The values of $M+M_0$ for each normal place are computed by (1), and the observations plotted. From a freehand curve drawn to represent them, the maximum and minimum velocities are

[1] *Report of the Chief Astronomer* for 1910, Appendix No. 2, pp. 147–149 (Ottawa: Government Printing Office, 1912).

read off to be $K+G=+48.0$, $G-K=-45.0$, whence $K=46.5$, $G=+1.5$. We then tabulate $\rho-G$, and compute $v+\omega$ by (3), and then $v-M+\omega-M_0$. The values of $v+\omega$ near 0° or 180° are uncertain, and are indicated by colons in the usual manner.

Plotting $v-M+\omega-M_0$ against $M+M_0$, we obtain the anomaly diagram shown in Fig. 2, in which the uncertain values are represented by the open circles. Superposing upon this the tracing-cloth on which are drawn the theoretical curves (shown in Fig. 1),

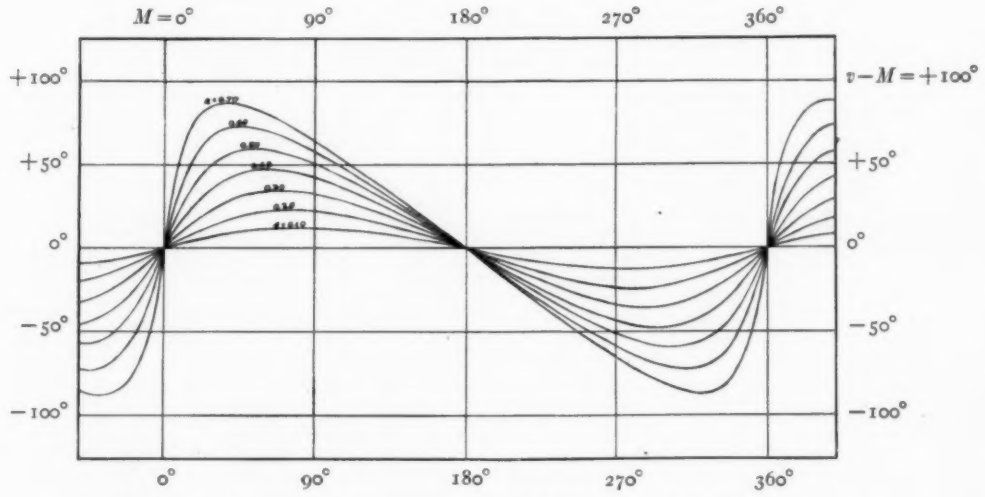


FIG. 1.—Curves giving $v-M$ as a function of M (tracing-cloth diagram)

we find that the curve for $e=0.40$ fits the observations very closely if it is shifted so that the periastron point upon it (the intersection of the steeper branches of the various curves) has the horizontal co-ordinate $-4°.5$ and the vertical co-ordinate $+18°$. But these quantities are respectively the values of M_0 and $\omega-M_0$; hence $M_0=-4°.5$ and $\omega=13°.5$. To find the remaining elements we then have $T=P\times M_0/360°$ and $\gamma=G-Ke\cos\omega$, which give $T=-0^d.32$ and $\gamma=-17.1$ km.

Even before solving these equations, we may compute the velocity-curve corresponding to these elements. Keeping the tracing-cloth in position above the anomaly diagram, we read off

from the latter the values of $v-M+\omega-M_0$ corresponding to $e=0.40$ and the tabular values of $M+M_0$. These are entered in the latter part of the table. From them the computed values of $v+\omega$ may be written down at once, and the corresponding values of $\rho-G$ read from a slide-rule. The residuals $O-C$ are given in the last column of the table.

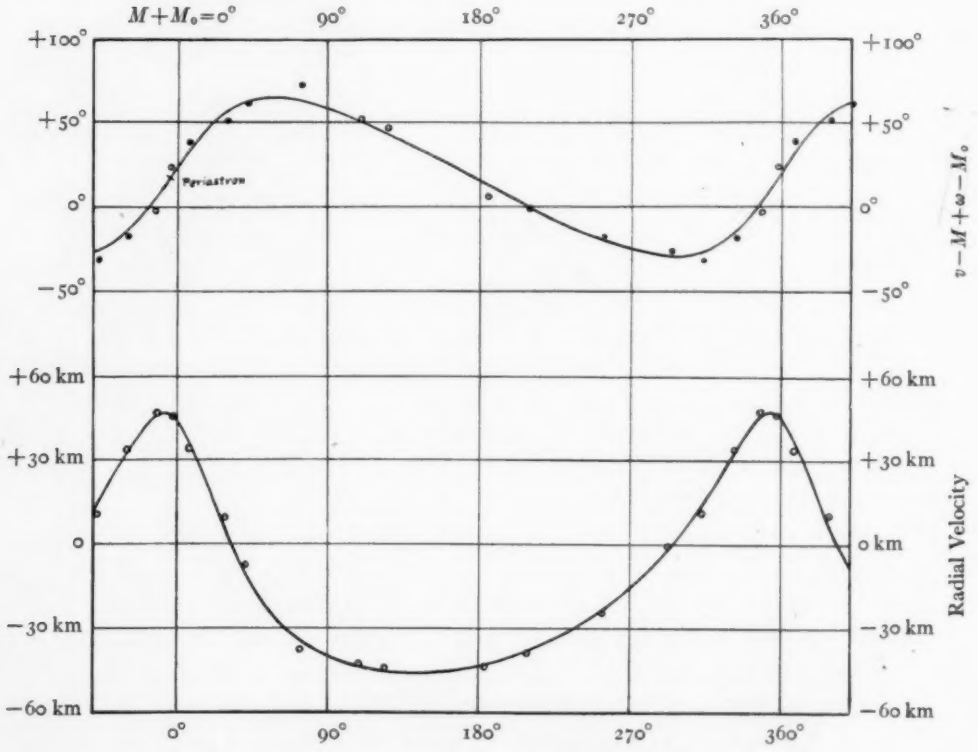


FIG. 2.—Anomaly diagram and velocity curve of α Draconis

From these residuals it appears that the representation of the observations by these elements, though already good, is capable of decided improvement. The residuals are negative near the maximum and minimum of the curve, and positive on the ascending and descending branches. Now the computed values at maximum and minimum can be altered only by changing G and

K, while it appears from the anomaly diagram that the points on the ascending and descending branches correspond to values of the equation of the center near its maxima, and are very sensitive to changes of e. A diminution of e, and therefore of the equation of the center, will diminish the values of v for points on the descending branch, and increase them for points on the ascending branch, and in both cases increase the computed velocity and raise the curve.

TABLE II

Data		Solution					From Curves		
Phase	Wt.	ρ	$M+M_0$	$\rho-G$	$v+\omega$	$\frac{v-M}{+\omega-M_0}$	$\frac{v-M}{+\omega-M_0}$	Comp. $\rho-G$	O−C
4.18....	1.5	+ 9.26	29°.3	+ 7.8	80°.3	+51.0	+57°.0	+ 3.0	+4.8
5.92....	5	− 8.61	41.5	−10.1	102.6	+61.1	+63.0	−11.6	+1.5
10.52....	4	−37.60	73.7	−39.1	147.2	+73.5	+62.4	−33.5	−5.6
15.65....	4	−43.00	109.7	−44.5	162.:	+52.:	+50.4	−43.8	−0.7
17.76....	4	−44.48	124.4	−46.0	172.:	+48.:	+44.	−45.6	−0.4
26.29....	4	−44.30	184.2	−45.8	190.:	+ 6.:	+13.6	−44.3	−1.5
29.89....	.2	−39.25	209.4	−40.7	208.9	− 0.5	+ 0.2	−40.5	−0.2
36.17....	2	−24.33	253.4	−25.8	236.3	−17.1	−20.0	−27.8	+2.0
41.87....	1.5	− 0.21	293.6	− 1.7	267.8	−25.8	−28.3	− 3.8	+2.1
44.56....	1.5	+10.62	312.1	+ 9.1	280.3	−31.8	−25.5	+13.3	−4.2
47.21....	4	+33.21	330.8	+31.7	313.0	−17.1	−13.8	+34.0	−2.3
49.53....	3	+46.20	347.0	+44.7	344.:	− 3.:	+ 4.8	+46.0	−1.3
50.86....	3	+45.30	356.3	+43.8	20.:	+24.:	+19.2	+44.9	−1.3
1.02....	1.5	+33.60	7.1	+32.1	46.2	+39.1	+33.8	+33.6	−1.5

The mean of the four residuals nearest the maximum velocity is −1.6, and that of the five residuals nearest the minimum is −1.7. We therefore, in beginning a second approximation, correct the previously assumed maximum and minimum by these amounts, and so find $G=-0.1$, $K=46.5$. We then compute new values of $v+\omega$, and construct a second anomaly diagram, in precisely the same way as before. The numerical data are given in Table III. When we superpose the tracing-cloth on this diagram, it appears at once that we must have approximately $M_0=0°$, $\omega-M_0=20°$, with e somewhat less than 0.40. A better determination of e may be made as follows. Keeping the tracing-cloth in such a position that $M=0°$, we shift it vertically until the curve for $e=0.40$ gives as good a representation as possible of the points near the top of the anomaly diagram. We then read from this

curve the maximum value of $v-M+\omega-M_0$, which is found to be $+64^\circ.8$. Displacing the tracing-cloth vertically until the best representation of the points near the bottom of the diagram is obtained, we read from the curve in its new position a minimum of $-26^\circ.2$. The mean of these, $+19^\circ.3$, gives an improved value of $\omega-M_0$, while half their difference is the maximum equation of the center, $45^\circ.5$, to which, by the table given earlier, corresponds an eccentricity of 0.390. If now we place the tracing-cloth so that the vertical co-ordinate of the periastron point corresponds to this value of $\omega-M_0$, we find easily that the best fit is obtained, on moving it horizontally, for $M_0=+1^\circ$.

TABLE III

$p \sin^2(v+\omega)$	$p-G$	$v+\omega$	$v-M+\omega-M_0$	M	v	Comp. $p-G$	O−C
1.4.....	+ 9.36	78°.4	+49°.1	+ 28°.3	+ 62°.4	+ 5.9	+3.5
4.8.....	− 8.51	100.6	+59.1	+ 40.5	+ 81.9	− 9.8	+1.3
1.4.....	−37.50	143.8	+70.1	+ 72.7	+117.4	−34.3	−3.2
0.6.....	−42.90	157.3	+45.6	+108.7	+143.1	−44.5	+1.6
0.3.....	−44.38	162.5:	+38.1:	+123.4	+151.7	−46.0	+1.6
0.3.....	−44.20	198.0:	+13.8:	−176.8	−178.5	−43.1	−1.1
0.5.....	−39.15	212.6	+ 3.2	−151.6	−166.3	−38.5	−0.7
1.5.....	−24.23	238.6	−14.8	−107.6	−142.5	−24.1	−0.1
1.5.....	− 0.11	269.9	−23.7	− 67.4	−112.7	− 1.9	+1.8
1.5.....	+10.72	283.3	−28.8	− 48.9	− 92.9	+13.9	−3.2
2.0.....	+33.31	315.8	−15.0	− 30.2	− 65.8	+32.6	+0.7
0.0.....	+46.30	357.::	+10.::	− 14.0	− 33.5	+45.3	+1.0
0.1.....	+45.40	12.5:	+16.3:	− 4.7	− 11.6	+46.0	−0.6
0.7.....	+33.70	43.5	+38.4	+ 6.1	+ 14.9	+38.0	−4.3

In making the compromises between the representation of different points, which are always necessary, it must be borne in mind that the relative weights of these points, *on the anomaly diagram*—that is, of the values of $v+\omega$ computed from the individual observations—are proportional to $p \sin^2(v+\omega)$ where p is the weight of the observed radial velocity. These need of course be computed only very roughly, and may usually be estimated mentally. Table III may be regarded as a continuation of Table II, and no unnecessary repetitions have been made in it. The latter part of this table gives the computation of the final velocity-curve, which has been made with the Allegheny tables. The values of M are found from those of $M+M_0$ given in Table II, and those of v taken

directly from the tables. The representation of the observations now appears to be satisfactory, and we proceed to calculate the final elements, in the same manner as in the first approximation.

The two sets of elements determined by the present method compare as shown in Table IV with the preliminary elements used by Harper, and the results of his least-squares solution. The first approximation by the present method is not so good as Harper's preliminary elements, which were obtained by King's graphical method (whether at the first trial or after more numerous attempts is not stated); but the second approximation gives results practically identical with those of the least-squares solution. Only one of the five quantities determined differs from the least-squares values by more than the probable error of the latter, and the weighted sum of the squares of the residuals is the same for both (within the accuracy of the reckoning).

TABLE IV

	Harper		Present Method	
	Preliminary	Final	1st Approx.	2d Approx.
P	51.38 d. (assumed throughout)	.....	.....	.....
K	47.0 km	46.25 ±0.24	46.5	46.5
e	0.41	0.384±0.003	0.40	0.390
ω	15°	19°.07 ±2°.21	13°.5	20°.3
T	0.00 d.	+ 0.284±0.230	− 0.32	+ 0.14
γ	−17.61 km	−17.03 ±0.15	−17.1	−17.1
$a \sin i$	.....	30,173,000 km	30,000,000	30,150,000
pvv	213.5	148.6	259.4	148.2

It would therefore appear that the new method is of very considerable precision. As regards its rapidity, an account of the time consumed was kept, showing that the first approximation, from the time of beginning to tabulate and plot the observations to the computation of the elements and residuals, required 78 minutes, and the second approximation 42 minutes more (not including the calculation of the sum of the squares of the residuals).

That these conclusions regarding the accuracy and rapidity of the method are generally applicable is shown by the results of several other test computations, for which an account of time was

kept. Some of the most difficult cases in the recent literature of the subject were purposely chosen. The results of previous computers have been derived by least squares in all cases except β Arietis, but in some instances the probable errors are not given. The normal places of the previous computers have been discussed independently by the present method, with the results given in Table V.

TABLE V

β ARIETIS

$P = 107.0$ Days

	Present Method	Ludendorff*
K	33.0 km	32.6
γ	− 0.5 km	− 0.6
e	0.88	0.88
ω	18°	19°.7
T	0.0 d.	+ 0.1
Time	63 min.	.....

* *Astrophysical Journal*, **25**, 324, 1907.

ϵ HERCULIS

$P = 4.0235$ Days

	Present Method	Baker†
K	71.25 km	70.39 ±1.08
γ	−24.0 km	−24.03 ±0.83
e	0.033	0.023±0.016
ω	180°	180° (assumed)
T	0.00 d.	−0.003±0.011
Time	51 min.	

† *Publications of the Allegheny Observatory*, **2**, 20, 1910.

θ TAURI

$P = 140.50$ Days

	1st Approx.	2d Approx.	Plaskett‡
K	28.5 km	29.5	29.1
γ	+43.7 km	+42.2	+42.9
e	0.67	0.70	0.694
ω	48°	50°	48.6
T	56.2 d.	56.6	56.12
Time	72 min.	59 min.	

‡ *Journal of the Royal Astronomical Society of Canada*, **6**, 231–239, 1912.

ι ORIONIS

$P = 29.136$ Days

	1st Approx.	2d Approx.	3d Approx.	Plaskett§
K	116 km	111.5	109.5	109.9 ±1.1
γ	+ 18.6 km	+ 23.0	+ 20.6	+ 21.3 ±0.86
e	0.77	0.75	0.75	0.754±0.005
ω	106°	110°	110°	113.3 ±1.0
T	+ 1.86 d.	+ 1.95	+ 1.91	+ 1.99 ±0.022
Time	2 hrs. 12 min.	Not noted, owing to interruptions		

§ *Astrophysical Journal*, **27**, 275, and **28**, 274, 1908.

B.D. −1°.1004

(Including secondary oscillation of same period as primary)

$P = 27.160$ Days

	1st Approx.	2d Approx.	Harper¶
K	94.5 km	95.5	93.04 ±2.2
γ	+26.5 km	+27.8	+26.12 ±0.7
e	0.80	0.77	0.765±0.007
ω	87°	87°	87°.03 ±2°.5
T	0.0 d.	+ 0.015	− 0.016±0.015
K'	.....	10.0 km	10.15 ±1.4
T'	.....	+13.58 d.	+12.32 ±1.3
Time	78 min.	68 min.	

¶ *Report of the Chief Astronomer*, 1910, Appendix 2, pp. 137–143.

The last three systems present problems of exceptional difficulty, and in the original discussions several successive least-squares solutions were necessary. Even in these cases, the present method has been found capable of giving solutions which agree very closely with the final values, with only an afternoon's work—probably less labor than was involved in finding the period and forming the normal places.

It would therefore appear probable that the new method may be of some utility to the practical computer.

PRINCETON UNIVERSITY OBSERVATORY
June 30, 1914

THE SPECTRA OF FOUR OF THE TEMPORARY STARS[1]

By W. S. ADAMS and F. G. PEASE

An important observation by Hartmann[2] in 1907 upon the spectrum of Nova Persei of 1901 showed that the principal nebular lines at λ 4960 and λ 5007, which were very prominent in the spectrum of the star when last observed during its brighter stages, had disappeared, and that the spectrum was essentially identical with that of the Wolf-Rayet star B.D.+35°4001. The interesting question thus raised, whether the disappearance of the principal nebular lines is characteristic of temporary stars in their later history, makes it desirable to secure spectra of such other Novae as can be obtained. Four such stars have been shown by the observations of Professor Barnard[3] to be of the fourteenth magnitude or brighter. With the magnitudes which he gives they are as follows:

	Magnitude
Nova Aurigae of 1891	14
Nova Persei of 1901	12.4
Nova Lacertae of 1910	12.5
Nova Geminorum No. 2 of 1912	10

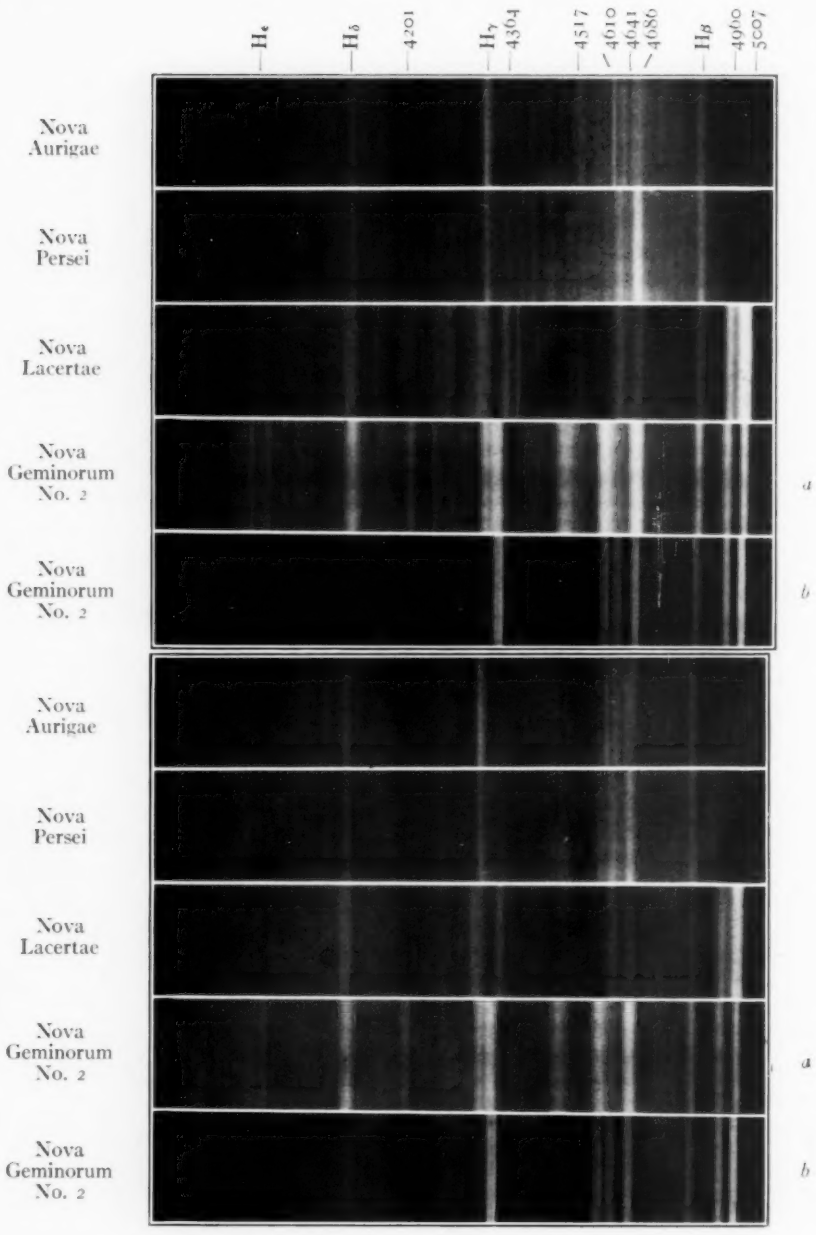
We have succeeded in obtaining photographs of the spectra of all of these stars with a small slit spectrograph at the primary focus of the 60-inch reflector. The spectrograph consists of a 60° prism of ultra-violet flint glass used with collimating and camera lenses of about 16 cm focal length by Steinheil. The instrument is mounted in the opening of the double-slide plate-holder, and after the star has been brought upon the slit the guiding is carried on as in the case of direct photography by keeping auxiliary stars upon the cross-wires of two eyepieces, one at either side of the field, the spectrograph being moved bodily by the guiding screws. A diagonal prism mounted in a tube with an eyepiece may be

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 87.

[2] *Astronomische Nachrichten*, **177**, 113, 1908.

[3] *Ibid.*, **194**, 401, 1913.

PLATE IX



UPPER REPRODUCTION FROM UNTOUCHED COPY OF WIDENED SPECTRA;
LOWER REPRODUCTION FROM A RETOUCHED PRINT

slipped back and forth behind the slit, and enables the observer to bring the star accurately upon the slit at the beginning of the exposure.

The photographs in the case of Nova Aurigae and Nova Lacertae required exposure times extending over more than one night. The observational data are given in Table I.

TABLE I

	Exposure Time	Date	Slit Width
Nova Aurigae	16 hours	1914, March 18–22	0.062 mm
Nova Persei	8 "	1913, November 27	0.044 "
Nova Lacertae	11.5 "	1913, October 29–31	0.044 "
Nova Geminorum No. 2	2 "	1914, February 22	0.044 "

Reproductions of the enlarged and widened spectra are shown in Plate IX. In addition a photograph of the spectrum of Nova Geminorum No. 2, taken with the Cassegrain spectrograph on October 12, 1913, is added for comparison. Owing to the small scale of the photographs and the consequent enlargement of grain, many false lines were introduced in the process of widening. Accordingly, two reproductions are shown. The first is a direct copy of the widened spectra. The second is from a print on which many of the false lines have been touched out by hand. Care has been taken in this case to leave untouched all parts of the spectrum in which true lines are seen on the original negatives. In addition, all of the real lines are marked on the reproductions.

The approximate intensities of the lines in the spectra are indicated in Table II upon a scale of 0 to 10. Probably several very faint lines are lost in the continuous spectrum which is particularly strong in the case of the first three stars. A dash in the table means that the line is not seen.

The single most interesting feature of these results is without doubt the absence of the chief nebular lines λ 4960 and λ 5007 from the spectra of Nova Aurigae and Nova Persei, and of the nebular line λ 4364, which is usually associated with them. The agreement of this observation in the case of Nova Persei with that of Hartmann in 1907 makes it appear that a spectrum in which these lines are

absent must be considered as probably the permanent spectrum of this star, and from the similar result for Nova Aurigae it seems reasonable to conclude that the absence of the nebular lines is characteristic of Novae in their later history. It is about twenty-three years since the discovery of Nova Aurigae and thirteen since that of Nova Persei, but only four and two in the cases of Nova Lacertae and Nova Geminorum No. 2.

TABLE II

	Nova Aurigae	Nova Persei	Nova Lacertae	Nova Geminorum No. 2 Plate IX*a*	Nova Geminorum No. 2 Plate IX*b*
$H\delta$	2	2	2	3	3
4201	–	Trace?	Trace	1	1
$H\gamma$	3	3	3	3	2
4364	–	–	Trace?	5	8
4517	–	Trace?	Trace?	2	2
4610	Trace	Trace	–	5	4
4641	1	1	3	Trace	3
4686	2	5	3	8	5
$H\beta$	2	2	2	3	3
4960	–	–	5	4	5
5007	–	–	10	7	8
Continuous spectrum	Very strong	Very strong	Very strong	Strong	Trace

The extraordinary intensity of the principal nebular line at λ 5007 in the spectrum of Nova Lacertae is its most important feature, but a peculiar fact in connection with it is the weakness or entire absence of the nebular line at λ 4364. These two lines have been ascribed by Professor Nicholson[1] to the same vibrating atom, and their opposite behavior in the spectrum of this star is certainly an unusual feature.

The three lines at λ 4610, λ 4641, and λ 4686 show remarkable differences of intensity in the different spectra, and even on the two photographs of Nova Geminorum there are large variations. Reference has been made by one of us in an earlier communication[2] to some of these differences in the case of Nova Geminorum. The first line of the principal series of hydrogen at λ 4686 seems to be especially subject to fluctuations, and it is interesting to note the

[1] *Monthly Notices, R.A.S.*, **74**, 488, 1914.

[2] *Ibid.*, **73**, 742, 1913.

great difference in its intensity in Nova Persei as compared with Nova Aurigae.

The intensity of the continuous spectrum of these stars appears to depend in general on their age, being greatest for the older Novae. The two photographs of Nova Geminorum show how very rapidly it may change, the earlier photograph showing hardly more than a trace of a continuous background.

As Hartmann has noted in the case of Nova Persei, and as the results show for both Nova Aurigae and Nova Persei, these spectra are essentially identical with the spectra of some of the Wolf-Rayet stars. With the disappearance of the principal nebular lines and the increase in intensity of the continuous spectrum the only marked points of difference have gone. This identity of spectrum, taken in connection with the well-known agreement of distribution relative to the Milky Way of Novae and Wolf-Rayet stars, makes it probable that at least a portion of the latter are temporary stars in the later stages of their history.

In the absence of knowledge as to the physical conditions under which the principal nebular lines are produced it is, no doubt, best to ascribe their disappearance to a change in these conditions in the radiating material. In view, however, of the considerable amount of evidence of various kinds which tends to support the hypothesis that the phenomenon of a temporary star is due to the star's entering a nebula, the suggestion that the disappearance of the chief nebular lines is coincident with the emergence of the star from the nebula may perhaps deserve some slight consideration.

MOUNT WILSON SOLAR OBSERVATORY
July 1, 1914

CONSTANT DIFFERENCES IN LINE-SPECTRA

By EMIL PAULSON

In all spectra there are lines or complexes of lines, between the wave-numbers of which the differences are constant. Kayser. however, considers these differences as "eine andere Art der Gesetzmässigkeit"; other authors such as Paschen regard them as but secondary products of the series.

However that may be, the knowledge of the constant differences constitutes the first step in the ordering of a complicated spectrum. When a number of constant differences have been found, an investigation of the Zeeman effect will shed more light on the affinity of the lines, and thus it will be possible to obtain the series.

The calculation of the constant differences of the wave-numbers is exceedingly difficult for a line-rich spectrum. Kayser[1] has made such a calculation with reference to the following spectra, namely: tin, lead, arsenic, antimony, bismuth, palladium, platinum, and ruthenium. Investigations of the same nature have also been made by Rydberg[2] and Snyder[3] on the spectra of argon and rhodium respectively.

In a recently published work[4] I have described a very simple mechanical means which, without any special difficulty, will facilitate the obtaining of the constant differences, particularly in line-rich spectra. In the work referred to above, the results of an examination of 40 spectra are given, which investigation has been made in the above-mentioned manner. Having had but a short time at my disposal, I had to confine the examination to the lines of the greatest intensities in each spectrum. This way I also consider the most practical. If the differences between the lines of the greatest intensities are known, it is easy to obtain the other lines having the same differences.

[1] *Handbuch der Spectroscopie*, **2**, 573; *Abh. der Berl. Akad.*, 1897.

[2] *Astrophysical Journal*, **6**, 338, 1897. [3] *Ibid.*, **14**, 179, 1901.

[4] *Beiträge zur Kenntnis der Linienspektren*. Diss. Lund, 1914.

In the following the results of some of the most minutely examined spectra are stated. As regards phosphorus, scandium, titanium, iron, cobalt, germanium, bromine, krypton, yttrium, and niobium, the results will be published in the *Annalen der Physik*.

The wave-lengths and intensities are naturally taken from the excellent tables in Professor Kayser's *Handbook*.[1] Where measurements of thousandths are given, these are used; the wave-length is then, however, reduced to hundredths. In other cases the very best measurements have been selected. The intensities are chiefly taken from Exner and Haschek.

The tables are written as follows: first the wave-lengths, then, in parentheses, the intensities, and lastly the wave-numbers and their differences.

1. CHLORINE

All the lines were examined. The wave-lengths and intensities are from Eder and Valenta. The results obtained are given in Table I.

TABLE I

5445.12 (1).....	18365.07	40.92	4469.57 (5).....	22373.53	108.02
5457.28 (3).....	18324.15		4491.25 (3).....	22265.51	
5444.41 (3).....	18367.46	40.33	4390.57 (3).....	22776.11	67.47
5456.39 (2).....	18327.13		4403.61 (5).....	22708.64	
4794.67 (10).....	20856.49	67.29	4323.52 (6).....	23129.30	108.09
4810.19 (9).....	20789.20	40.72	4343.82 (10).....	23021.21	
4819.63 (9).....	20748.48		4291.86 (5).....	23299.92	66.86
			4304.21 (4).....	23233.06	
4601.19 (4).....	21733.51	108.29			
4624.23 (3).....	21625.22		4234.14 (5).....	23617.56	40.66
			4241.44 (8).....	23576.90	67.02
4497.45 (½).....	22234.82	107.99	4253.53 (9).....	23509.88	
4519.4 (½).....	22126.83				

[1] *Handbuch der Spectroscopie*, **5**, 6.

TABLE I—*Continued*

4124.15 (1).....	24247.40	40.14	3845.55 (8).....	26004.08	40.37
4130.99 (4).....	24207.26		3851.53 (8).....	25963.71	
3848.03 (2).....	25987.30	40.23	3833.50 (8).....	26085.81	67.11
3854.00 (4).....	25947.07		3843.39 (5).....	26018.70	39.94
3845.83 (8).....	26002.22	66.84	3849.30 (2).....	25978.76	
3855.74 (2).....	25935.38				

2. MANGANESE

The number of lines examined here amounts to 42. The wave-lengths and intensities of the red lines are from Fritsch. The lines presented in Table II form pairs. The difference 173.69 is also observed by Kayser.[1] In conjunction with a third line the pairs form two triplets, which are the first lines of the two secondary series. I have observed only the pairs, the third line not being included in the investigation on account of its slight intensity.

TABLE II

6013.72 (10)	16628.64	8.68	4783.61 (10 R) ...	20904.72	173.71
6016.86 (10)	16619.96	14.24	4823.69 (10 R) ...	20731.01	
6022.02 (10)	16605.72		3547.94 (10 R) ...	28185.37	173.77
4030.92 (100 R) ...	24808.24	14.27	3569.95 (10 R) ...	28011.60	
4033.24 (100 R) ...	24793.97	8.67			
4034.65 (50 R)	24785.30		5341.18 (8)	18722.46	230.04
			5407.62 (6)	18492.42	
5470.83 (7)	18278.76	35.95	4018.28 (10)	24886.27	229.61
5481.61 (6)	18242.81		4055.70 (10)	24656.66	
2798.36 (50 R)	35735.21	35.97	3578.03 (10)	27948.34	229.85
2801.18 (50 R)	35699.24		3607.70 (6)	27718.49	

[1] *Handbuch*, 2, 551.

3. NICKEL

Number of the lines examined, 51. The wave-lengths and intensities of the red lines are taken from Stötting. First there was found a group of 4 lines, which was twice completely repeated, once incompletely. These are given in Table III, also the two pairs, at the foot of the table.

TABLE III

6191.39 (10)	16151.47	
		168.27
6256.57 (10)	15983.20	
		179.98
6327.82 (6)	15803.22	
3472.71 (10)	28795.96	
		168.17
3493.11 (30)	28627.79	
		179.99
3515.21 (30)	28447.80	
		751.65
3610.61 (10)	27696.15	
3393.10 (20 R)	29471.57	
		348.81
3433.74 (15)	29122.76	
		751.40
3524.68 (50)	28371.36	
5035.55 (10)	19858.80	
		2889.80
5893.10 (10)	16969.00	
3134.22 (30)	31905.87	
		2890.09
3446.40 (30 R)	29015.78	

4. MOLYBDENUM

Among the numerous lines of this spectrum only a few are intense. My examination has been extended to 50 lines and has given the result shown in Table IV.

TABLE IV

4610.06 (10)	21691.69	448.72	3158.28 (10 R) ...	31662.81	121.64
4707.44 (3)*	21242.97		3170.46 (20 R) ...	31541.17	
4326.29 (8)	23114.49	448.31	2775.74 (10)	36026.43	121.08
4411.86 (20)	22666.18		2785.10 (8)	35905.35	
3798.41 (50 R) ...	26326.80	448.89	4149.73 (8)	24097.96	719.77
3864.30 (50 R) ...	25877.91		4277.49 (10)	23378.19	
5650.40 (4)	17697.86	121.29	4107.68 (8)	24344.65	719.74
5689.39 (4)	17576.57		4232.82 (10)	23624.91	
5533.26 (6)	18072.52	121.42	3363.94 (8)	29727.05	718.84
5570.69 (6)	17951.10		3447.30 (10)	29008.21	

*Hasselberg.

5. PALLADIUM

In this case all the lines in the arc spectrum and also some in the spark spectrum, in all 348, were examined. The intensities are taken from Kayser, whose measurements of the wave-lengths have also been used. Table V gives the differences discovered.

Later on I found that 6 of these pairs have also been observed by Kayser. A third line, however, belongs to the pairs, so that they in reality form two triplets. This fact I had not observed, my examination not being extended to so great intervals. Two other constant differences were discovered. The results of this work are given in Table VI.

TABLE V

4817.66 (9)	20756.96	1191.11	3002.78 (4 R)	33302.47	1191.08
5110.94 (6)	19565.85		3114.16 (5 R)	32111.39	
3634.84 (10 R) ...	27511.53	1191.10	2922.62 (7 R)	34215.88	1191.11
3799.33 (5 R)	26320.43		3028.03 (4 R)	33024.77	
3460.89 (7 R)	28894.29	1191.24	2577.20 (1)	38801.80	1191.13
3609.71 (9 R)	27703.05		2658.82 (2)	37610.67	
3287.38 (5)	30419.36	1191.30	2433.19 (2)	41098.31	1190.90
3421.37 (8 R)	29228.06		2505.80 (2)	39907.41	
3242.82 (10 R) ...	30837.36	1191.39	2254.40 (1)	44357.70	1190.99
3373.14 (6 R)	29645.97		2316.60 (0)	43166.71	

TABLE VI

4875.58 (7)	20510.38	1627.60	3251.75 (5 R)	30752.66	1628.54
5295.83 (10)	18882.78		3433.58 (5 R)	29124.12	
4170.01 (5)	23980.76	1628.25	2461.41 (1 U)	40627.12	1628.60
4473.77 (6)	22352.51		2564.2 (1 U)	38998.52	
3832.45 (10)	26092.97	1628.26	5529.66 (6 u).....	18084.29	402.79
4087.52 (6)	24464.71		5655.63 (5 u).....	17681.50	
3719.06 (4 R)	26888.52	1628.21	5497.06 (2)	18191.54	402.75
3958.78 (5 R)	25260.31		5621.52 (2)	17788.79	
3481.31 (7 R)	28724.82	1628.15	3441.55 (6 R)	29056.68	402.81
3690.49 (6 R)	27096.67		3489.93 (4 R)	28653.87	
3302.25 (6 R)	30282.38	1628.51	3258.91 (6 R)	30685.11	402.73
3489.93 (4 R)	28653.87		3302.25 (6 R)	30282.38	
3258.91 (6 R)	30685.11	1628.43	3028.03 (4 R)	33024.77	402.81
3441.55 (6 R)	29056.68		3065.42 (4 R)	32621.96	

6. LANTHANUM

Only 25 strong lines were examined in which were found the pairs shown in Table VII.

TABLE VII

4238.56 (20)	23592.91	519.16	4077.49 (10)	24524.89	381.21
4333.93 (20)	23073.75		4141.87 (10)	143.68	
3949.24 (20)	25321.32	518.81	3988.67 (15)	25071.01	381.11
4031.85 (20)	24802.51		4050.24 (10)	24689.90	

7. SAMARIUM

In this line-rich spectrum the 48 strongest lines were examined. The results obtained were as presented in Table VIII.

TABLE VIII

4424.55 (20)	22601.37	173.31	4391.03 (10)	22773.70	929.53
4458.70 (10)	22428.06	421.60	4577.88 (10)	21844.17	
4544.12 (10)	22006.46	615.03	4152.38 (10)	24082.58	929.72
4674.77 (10)	21391.43		4319.12 (10)	23152.86	
4225.48 (10)	23665.95	172.69	4454.84 (10)	22447.50	548.31
4256.54 (10)	23493.26	421.59	4566.38 (10)	21899.19	
4334.32 (10)	23071.67	614.49	4329.21 (10)	23098.91	548.55
4452.92 (10)	22457.18		4434.52 (20)	22550.36	

8. EUROPIUM

First, 4 pairs of lines with a very great intensity were discovered, the oscillation differences of which in the mean are 1669.71. Table IX shows the results of this work.

TABLE IX

4205.20 (100)	23780.08	1669.88
4522.80 (20)	22110.20	
4129.90 (100)	24213.66	1669.50
4435.74 (50)	22544.16	
3725.10 (30)	26844.92	1669.70
3972.16 (50)	25175.22	
3688.57 (20)	27110.77	1669.75
3930.66 (50)	25441.02	

I have more recently examined all the lines in the arc spectrum, 280 in all, in order to ascertain how often this difference occurs in the whole spectrum. Table X gives the pairs with the same difference which were then also discovered.

TABLE X

3744.35 (2)	26706.90	1669.66	2820.90 (4)	35449.68	1669.78
3994.05 (1)	25037.24		2960.34 (3)	33779.90	
3687.93 (1)	27115.48	1670.19	2729.46 (5)	36637.29	1669.69
3930.00 (2)	25445.29		2859.79 (3)	34967.60	
3678.41 (1)	27185.66	1670.18	2705.36 (2)	36963.66	1669.88
3919.19 (1)	25515.48		2833.36 (2)	35293.78	
3511.20 (3)	28480.29	1669.56	2701.21 (3)	37020.44	1669.89
3729.85 (2)	26810.73		2828.81 (4)	35350.55	
2906.80 (5)	34402.09	1669.62	2685.74 (3)	37233.68	1670.03
3055.07 (4)	32732.47		2811.86 (2)	35563.65	
2862.69 (3)	34932.18	1669.70	2559.30 (1)	39073.18	1669.59
3006.39 (2)	33262.48		2673.54 (3)	37403.59	

It is here, as often happens, a question of a complex of lines, the wave-numbers of which have constant differences in the horizontal and vertical rows. Two differences in question are set forth in Table XI.

TABLE XI

4435.74 (50)	22544.16	433.96	4383.36 (3)	22813.55	265.67
4522.80 (20)	22110.20		4435.01 (2)	22547.88	
4181.06 (2)	23917.38	433.72	4182.42 (4)	23909.61	265.98
4258.28 (2)	23483.66		4229.47 (1)	23643.63	
4129.90 (100)	24213.66	433.58	3930.66 (50)	25441.02	265.80
4205.20 (100)	23780.08		3972.16 (50)	25175.22	
3943.21 (2)	25360.05	433.89	3916.93 (2)	25530.19	265.16
4011.85 (3)	24926.16		3958.04 (1)	25265.03	
3425.19 (3)	29195.46	433.13	3688.57 (20)	27110.77	265.85
3476.77 (1)	28762.33		3725.10 (30)	26844.92	
			3213.84 (3)	31115.42	265.98
			3241.55 (3)	30849.44	

9. CERIUM

The examination has included the 30 strongest lines and has given the results presented in Table XII.

TABLE XII

4562.55 (8)	21917.57	78.55	4367.15 (4)	22898.23	1131.27
78.96 (5)	839.02		4594.12 (4)	21766.96	
4471.38 (5)	22364.46	78.14	4306.89 (4)	23218.61	1131.42
87.06 (5)	286.32		4527.51 (5)	22087.19	
4152.17 (9)	24083.71	78.49	4246.07 (8)	23551.19	1131.37
65.76 (10)	005.22		4460.34 (8)	22419.82	
4150.11 (10)	24095.74	78.52	4186.76 (10)	23884.82	1131.43
63.68 (6)	017.22		4394.95 (4)	22753.39	
			3952.75 (8)	25298.84	1131.34
			4137.79 (9)	24167.50	

10. XENON

In the first spectrum of this element the triplets given in Table XIII have been discovered (measurements are taken from Baly).

TABLE XIII

4658.94 (1)	21464.11	759.62	3985.39 (3)	25091.64	3684.87
4829.87 (4)	20704.49	3684.08	4671.42 (10)	21406.77	760.29
5875.30 (1)	17020.41		4843.44 (2)	20646.48	
4109.84 (5)	24331.84	3685.36	3951.16 (10)	25309.03	3684.88
4843.44 (2)	20646.48	759.52	4624.46 (15)	21624.15	759.39
5028.42 (2)	19886.96		4792.77 (1)	20864.76	

11. IODINE

An investigation of 50 lines has resulted in the pairs given in Table XIV (measurements are taken from Konen).

TABLE XIV

5739.61 (6)	17422.79	107.09	4221.24 (10)	23689.73	958.07
75.11 (6)	315.70		4399.15 (8)	22731.66	
5464.60 (10)	18299.60	107.83	3686.79 (8)	27123.86	956.95
96.99 (10)	191.77		3821.62 (8) ?.....	26166.91	
4423.98 (8)	22604.08	107.09	4512.80 (8)	22159.19	1355.59
45.04 (8)	496.99		4806.86 (10)	20803.60	
4342.25 (8)	23029.53	107.53	3561.32 (8)	28078.47	1356.00
62.62 (6)	922.00		3742.03 (8)	26723.47	
5338.31 (8)	18732.52	957.46	3512.82 (6)	28467.16	1355.72
5625.86 (10)	17775.06		3688.48 (8)	27111.44	
4292.14 (6)	23298.40	958.12			
4476.22 (6)	22340.28				

12. DYSPROSIUM

Among the 25 lines examined the pairs shown in Table XV were found.

TABLE XV

4111.50 (8)	24322.02	828.54	4050.73 (9)	24686.91	136.67
4256.50 (8)	23493.48		4073.28 (10)	24550.24	
3968.55 (10)	25198.12	828.39	3978.70 (10)	25133.84	137.52
4103.45 (9)	24369.73		4000.59 (10)	24996.32	
3944.83 (10)	25349.64	828.48			
4078.11 (10)	24521.16				

13. ERBIUM

There were twenty lines examined with results as presented in Table XVI.

TABLE XVI

4301.78 (8)	23246.19	440.29
4384.83 (8)	22805.90	
3938.79 (10)	25388.51	440.34
4008.31 (8)	24948.17	
3830.69 (10)	26104.96	440.24
3896.40 (15)	25664.72	

14. THULIUM

There were 30 lines examined with the results embodied in Table XVII.

TABLE XVII

3751.98 (5)	26652.59	71.62	3958.21 (8)	25263.95	243.00 / 666.28 } 909.28
3762.09 (20)	26580.97		3996.65 (5)	25020.95	
3734.29 (15)	26778.86	71.02	4105.99 (15)	24354.67	
3747.22 (20)	26707.84				
			3718.07 (20)	26895.68	
3453.82 (15)	28953.45	71.50	3751.98 (5)	26652.59	243.09 / 665.94 } 909.03
3462.37 (20)	28881.95		3848.13 (50)	25986.65	
3700.41 (15)	27024.04	245.18			
3734.29 (15)	26778.86				
3668.24 (5)	27261.03	245.25			
3701.54 (15)	27015.78				

15. LUTETIUM

All the lines in the arc spectrum, 48 in all, were included in the investigation, which has given the results shown in Table XVIII.

TABLE XVIII

3876.82 (2)	25794.34	5771.55	2754.29 (2)	36306.99	1764.31
4994.31 (3)	20022.79	1764.39	2894.97 (3)	34542.68	
5476.93 (6)	18258.40		3118.56 (4)	32066.08	2630.51
2613.50 (1)	38262.87	5771.29	3397.25 (5)	29435.57	639.45
3077.72 (4)	32491.59	1764.43	3472.69 (6)	28796.12	
3254.45 (4)	30727.16	2216.83	2798.34 (1)	35735.47	2630.89
3507.50 (3)	28510.33		3020.73 (3)	33104.58	639.79
2900.44 (3)	34477.52	1764.21	3080.26 (2)	32464.79	
3056.86 (2)	32713.31	2217.23	2963.46 (3)	33744.34	639.76
3279.11 (2)	30496.08		3020.73 (3)	33104.58	

LUND, SWEDEN
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TELESCOPIC VISION OF AN ILLUMINATED SURFACE

By FRED W. VORHIES

INTRODUCTION

An interesting aspect of the subject of telescopic vision has been discussed in a series of articles written by Stoney.[1] In these articles Stoney discusses the subject from a theoretical standpoint but makes some suggestions as to how apparatus could be set up and used in verifying his theories and assumptions. It is the purpose of this paper to show how this apparatus works in practice and to give the results obtained by its use. In view of the fact that there has been so much discussion during the past, concerning the ability to see details upon such astronomical objects as Mars, the author trusts that this contribution will be of value. Reference should be made to Stoney's articles for the theory of his experiments, for it is too long to reproduce in full here.

As a basis for his theories Stoney makes the following assumptions which I adopted and proved to be correct:

(1) In studying an illuminated surface or object—the planet Mars, for example—the real object, No. 1, can be removed and a substitute, No. 2, can be used in its stead. No. 2 is to be a transparent object with the details of No. 1 delineated upon it.

(2) When No. 2 is illuminated from behind with diffused light, the same sort of image will be formed in the telescope as when No. 1 is illuminated from in front with diffused light.

(3) A lens can be placed immediately in front of No. 2 and the image which it casts of a point-source of light, placed at some distance (the light from this source passing through object No. 2), is called the concentration image.

(4) The introduction of this lens does not alter the optical conditions.

(5) One method of analysis permits us to consider that the ordinary telescopic images of illuminated surfaces are made up of an infinite number of the above concentration or partial images.

[1] *Philosophical Magazine*, **16**, 318, 796, 950, 1908.

The above assumptions enable us to reduce the highly complex problem of image-formation by diffused light to a simple diffraction problem. It should further enable us to place, at least, an inferior limit to the image-forming power of our telescopes.

EXPERIMENTAL METHODS AND THEORY

Fig. 1 is a diagram of the experimental apparatus in its final form. M is a monochromatic illuminator which gets its light from an arc lamp. Light from M is focused by the lens L_1 on the pin-hole S, which acts as the point-source mentioned above, that illuminates the object (called No. 2 above) O. Behind and touching O is the achromatic lens L_2 which focuses the light from S upon the aperture A which is made separate from the camera C. This focused image of the point-source with its accompanying diffraction pattern, caused by the object O, is the concentration image which has been mentioned above. The camera represents our telescope and the aperture A represents the rim of the object glass.

FIG. 1

The object O was made by drilling a hole 4.5 cm in diameter in a plate of copper. Across this, wire was placed, varying from 0.08 mm to 0.62 mm in diameter. This wire was placed in such a manner as to form details which could be studied in a photograph. The distance OA is determined by the focal length of the lens L_2. In this case OA was 120 cm.

If we now suppose that the object O is to represent the planet Mars, we can obtain by means of Airy's formula,[1] which gives the diameter of the diffraction rings formed at A by a circular aperture of given size at O, when illuminated by a point-source at S, the size of the apertures that must be used at A to represent given

[1] *Wood's Physical Optics*, 2d ed., p. 237.

telescope objectives. Further, as I shall now show, the distance from O to A, the size of the object O, and the size of the aperture A can be so chosen as to give on the photographic plate an image of the same optical excellence as that obtained by a telescope of given size at a given distance from a heavenly body which is represented by the object O. In other words we can duplicate astronomical conditions in the laboratory.[1]

In order to use Airy's formula, which refers to diffraction rings formed with a point-source of light, and apply it to an astronomical object seen by reflected diffused light, use must be made of the five assumptions made in the first part of this paper. Airy's formula for the fifth maximum diffraction ring is $\sin\theta=\frac{2.361}{\gamma}\lambda$, where θ is the angle of diffraction, λ the wave-length of the light used, and r the radius of the lens casting an image of the point. In this work light of wave-length 0.000055 cm was used, and r, the radius of the planet Mars, was taken as 2,100 miles. Then $\sin\theta=\frac{2.361\times 0.000055}{2100}$. Again remembering, according to the assumptions, that the lens is immediately in front of the body (No. 2) substituted for the actual planet (No. 1), we get another expression for $\sin\theta$ which is given by the radius of the fifth diffraction ring divided by the distance, say 35,050,000 miles, from the earth to the planet Mars. From these two expressions the value of x is found to be 2.138 cm. $2x=4.276$ cm, the diameter of the fifth maximum ring in the diffraction pattern from the object representing Mars.

According to Stoney's theory it is the relation of the aperture of the telescope to the diameter of the diffraction pattern that determines the excellence of the image. Perhaps it would seem from this that a telescope 4.28 cm in diameter should give an image of the planet. If we were dealing with ideal conditions, a telescope of this size might possibly give an outline of the planet, but the image would contain no details. It must be remembered that in nature we have an overlapping of a great many partial

[1] Optical conditions only have been duplicated. The assumption was made that the details on the planet Mars contrast against the surface as well as dark wires would against white paper.

images, and that the small telescope would not admit enough of these various patterns to form a satisfactory image.

We shall designate by the constant K the ratio of the diameter of the aperture of the telescope to the diameter of, say, the fifth bright ring in this diffraction pattern. If a 24-inch telescope is used, the value of K is found to be $\frac{24\times 2.54}{4.276}$, or 14.27. This means that the diameter of the telescope is 14.27 times greater than the diameter of the fifth bright diffraction ring from Mars. This value of K we shall now use in determining the diameter of A, the aperture in the laboratory apparatus that is to replace the actual objective aperture of the telescope considered.

If the diameter of O is 4.5 cm and OA equals 120 cm, $\sin\theta = \frac{2.361\times 0.000055}{2.25}$

Also as above, $\sin\theta = \frac{x}{120}$

Eliminating $\sin\theta$, $x = .006928$ cm,

$2x = .013856$ cm, the diameter of the fifth bright diffraction ring from O.

$.013856\times 14.27 = .1976$ cm, or about 2 mm, which is used as the diameter of the aperture which represents a 24-inch telescope in the experimental apparatus. Using this relationship, the sizes of the other apertures to be used are easily determined.

Table I shows the sizes of the apertures used and the diameters of the telescopes which these apertures represent in the experimental apparatus.

TABLE I

Aperture	Telescope
4 mm	48 inch
3	36
2	24
1.25	15
1	12
.6	7.2
.4	4.8

Returning now to the object of O, if 4.5 cm equals 4200 miles, the wires on the object which were 0.62 mm, 0.38 mm, and

0.08 mm, respectively, would equal 58 miles, 35.5 miles, and 7.5 miles respectively, on the planet Mars. These calculations were made because a great deal of discussion has taken place among astronomers concerning certain details upon the surface of the planet. So-called canals varying in width from 60 miles to 20 miles have been viewed by different astronomers. In order to investigate this point, I made the smallest details represent objects 7.5 miles in width. The pattern formed by these wires was chosen quite at random and in no way is supposed to represent the map of Mars. The things that are faithfully represented are the disk of Mars at this given distance, and dark objects of the sizes specified.

If an incandescent lamp behind a ground-glass screen were placed at *S*, instead of the point-source of light, there would be an infinite number of point-sources. Each of these sources or points would be capable of illuminating *O* and would thus give its diffraction pattern at *A*, or, as we have said, a concentration image. The advantage, then, in using the point-source *S* is simply to cut down the complexity of the diffraction pattern at *A* and in turn to simplify the image produced in the camera *C*. Then again, if this diffused white light were used, there would be a large number of different wave-lengths in operation. This is simplified by using the illuminator *M*. By the use of these two devices the image is greatly simplified and a study of it is much more easily made.

Upon examining the diffraction pattern at *A*, it is found that it consists not only of the usual bright central spot surrounded by bright and dark rings caused by the circular form of the object that represents Mars, but also in the field there are bright and dark patches which are spectra caused by the wires used to represent superficial details on the planet. Now any part of this pattern, when admitted into the camera or telescope, is capable of giving some kind of an image of the object *O*. This fact permits one more step in the analysis and gives a means of obtaining partial images on the sensitive plate. Under ordinary conditions most of the light is contained in the central bright spot and the first five or six bright rings. This being true, if this part of the pattern is admitted into the telescope, a good image should be obtained. If this part of the pattern does not enter the telescope, the image

will not be so good, the quality depending on the particular part of the pattern admitted. It is an interesting fact that if only the central maximum is admitted by the telescope aperture, we get no image of the object, although a large amount of light will be admitted. This is in conformity with Abbe's theory of image-formation.[1]

The above discussion now permits the insertion of two important definitions. The image obtained on the sensitive plate when the diffraction pattern from a point-source is admitted into the telescope or camera is called a partial image. If the central bright spot falls symmetrically within the aperture of the camera objective, an optimum partial image is formed. This seems to be the best image that it is possible to obtain of a given object with a given aperture, and is theoretically better than any image of a similar object taken under similar conditions with diffused light. In other words, with this laboratory apparatus we not only can imitate astronomical conditions, but can obtain even better images than are possible under natural conditions, because we can control the number of partial images that go to make up the final image. If the aperture A is moved to one side, the central bright spot will not be admitted into the camera and the image obtained will be formed by light from some part of the outer diffraction pattern. This image is one of the inferior images which, along with all the other partial images, good and inferior, go to make up the natural image of the object. From this it is easily seen that telescope images of bodies illuminated by diffused light are simply the result of an infinite number of partial images, and that instead of having one diffraction pattern falling within the camera, there is an infinite number of these patterns. It is then quite obvious that a natural image cannot be as good as an optimum image, because the natural image can be considered to be made up of the optimum and a number of other images which are very inferior. As the experimental results show, a natural image may approach the optimum in excellence but it cannot equal or surpass it.

EXPERIMENTAL RESULTS

The experimental results of this work are found in the accompaning photographs. Plate X contains four series of partial images

[1] *Wood's Physical Optics*, 2d ed., p. 223.

taken with different sized apertures. Series 1 was taken with a 2-mm aperture and represents the partial images that go to make up the actual image that would be seen with a 24-inch telescope. No. 1 was taken with the central bright spot of the diffraction pattern focused in the center of the aperture. This is what we call an optimum image and is better than any that can be obtained in actual practice under the same conditions. Nos. 2–8 were taken by moving the aperture of the camera to one side by means of the micrometer screw. The aperture was moved one half-millimeter each time. This means that No. 8 was produced by that part of the diffraction pattern 3.5 mm out from the central bright spot. Now it can be seen that in actuality there would be an infinite number of these partial images, and that the images we obtain with optical instruments are produced by an infinite number of these partials, interfering as they are superposed by a lens or by the eye. The increasing inferiority of the partial images as we go from 1 to 8 is to be noted.

The phenomenon of the change in appearance of the lines from black to white can be accounted for by the laws of diffraction. It will be noticed in the last part of each series that the horizontal wires almost disappear. This is caused by the movement of the aperture *A* to one side. After it has been moved 2 or 3 mm, it is in such a position that the parts of the diffraction pattern that enter the camera are altogether those caused by the perpendicular wires. This part of the pattern is not capable, therefore, of giving images of the horizontal wires. Investigations on a similar problem have been made with ordinary wire screens.[1]

No. 9 is what we might call the composite image. It is to be taken as an approximate imitation of a natural image. It is obtained by superposing the exposures 1–8 on the same plate, giving equal times of exposure to all eight. From the discussion above the conclusion would be drawn that this image, No. 9, should not be as good as No. 1. In series 1 a very little difference can be seen between 1 and 9. This is because the excellence and large number of the good images in the series outweigh the defects contributed by the poorer images. This point is made more clear by the following series. Series 2, 3, and 4 were taken exactly as

[1] A. B. Porter, *Philosophical Magazine*, 11, 154, 1906.

series 1, with exactly the same lateral movement of the aperture A, and show the effects of smaller apertures. Series 2 was taken with a 1.25-mm aperture representing a 15-inch telescope, series 3 with a 1-mm aperture representing a 12-inch telescope, and series 4 with a 0.6-mm aperture representing a 7.2-inch telescope.

It will be noted that as the aperture decreases in diameter, the number of good images decreases and that No. 9, the composite image, gradually gets poorer than the optimum No. 1. In series 4 the optimum itself is not distinct. This is on account of the fact that the aperture was so small that the main part of the diffraction pattern was not admitted. The subject of partial images will be discussed again in connection with Plate XII.

Series 1 of Plate XI shows eight optimum images represented by Nos. 1–8 inclusive, obtained with apertures of the sizes given in Table II.

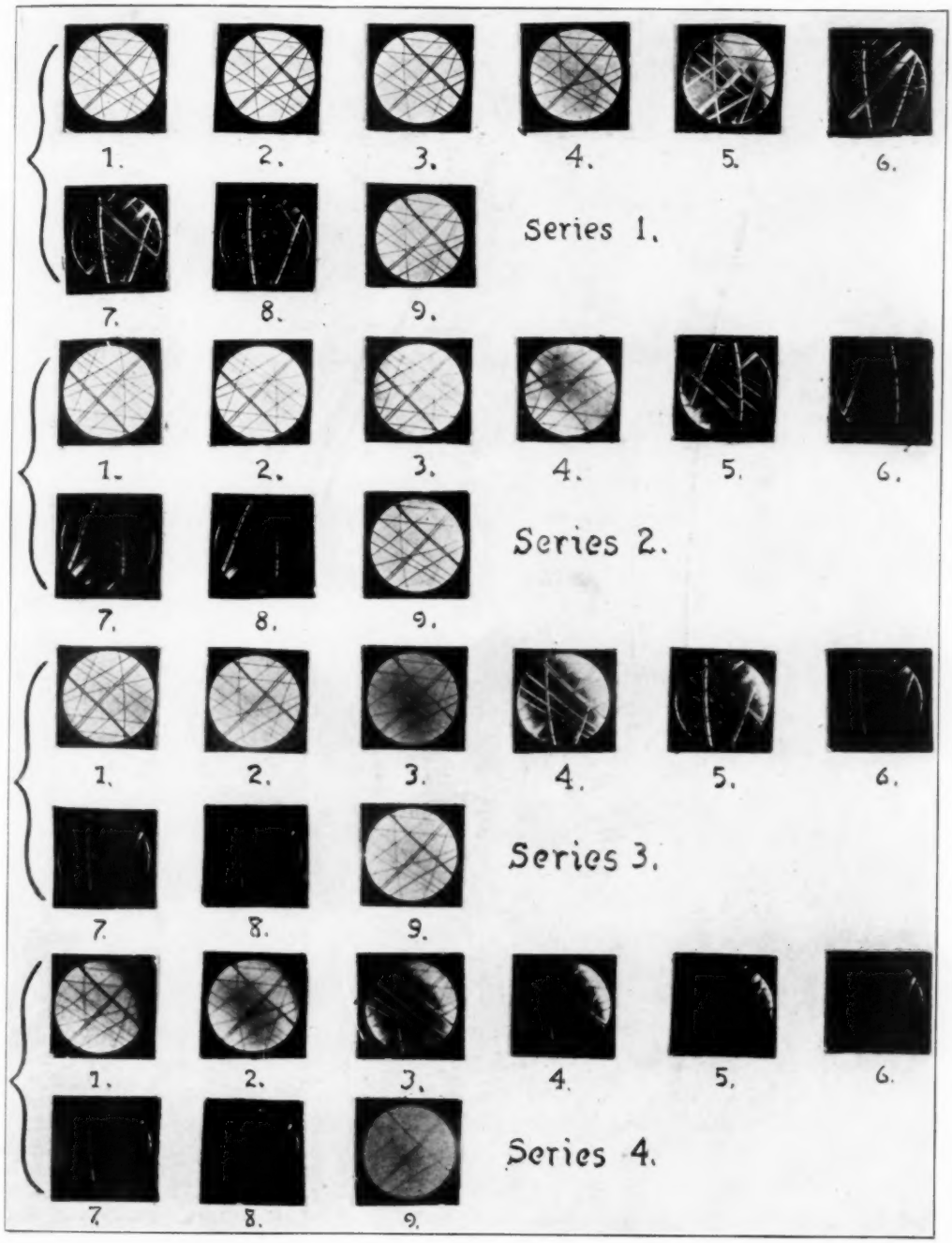
TABLE II

No.	Aperture	Telescope
1	4 mm	48 in.
2	3	36
3	2	24
4	1.25	15
5	1	12
6	0.6	7.2
7	0.4	4.8
8	0.3	3.6

Series 2 is taken with the same apertures and under the same conditions except that the object O was illuminated with diffused light instead of monochromatic light. This was done by removing the point-source S (Fig. 1) and placing in its stead an incandescent light in front of which was a ground glass. This series represents, then, the actual telescope images.

Series 3, Plate XI, was taken with the same apertures as were used in series 1 and 2, but with reflected light. To accomplish this a white cardboard was placed just back of object O and directly against the wires. Four incandescent lights were then placed in front and a little to one side of the object. The lens L_2 was removed while this series was being taken, so that the illumination on the object would be more intense. Nos. 7 and 8 are missing in series 3,

PLATE X


1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 1.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 2.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 3.
1. 2. 3. 4. 5. 6.
7. 8. 9.
Series 4.

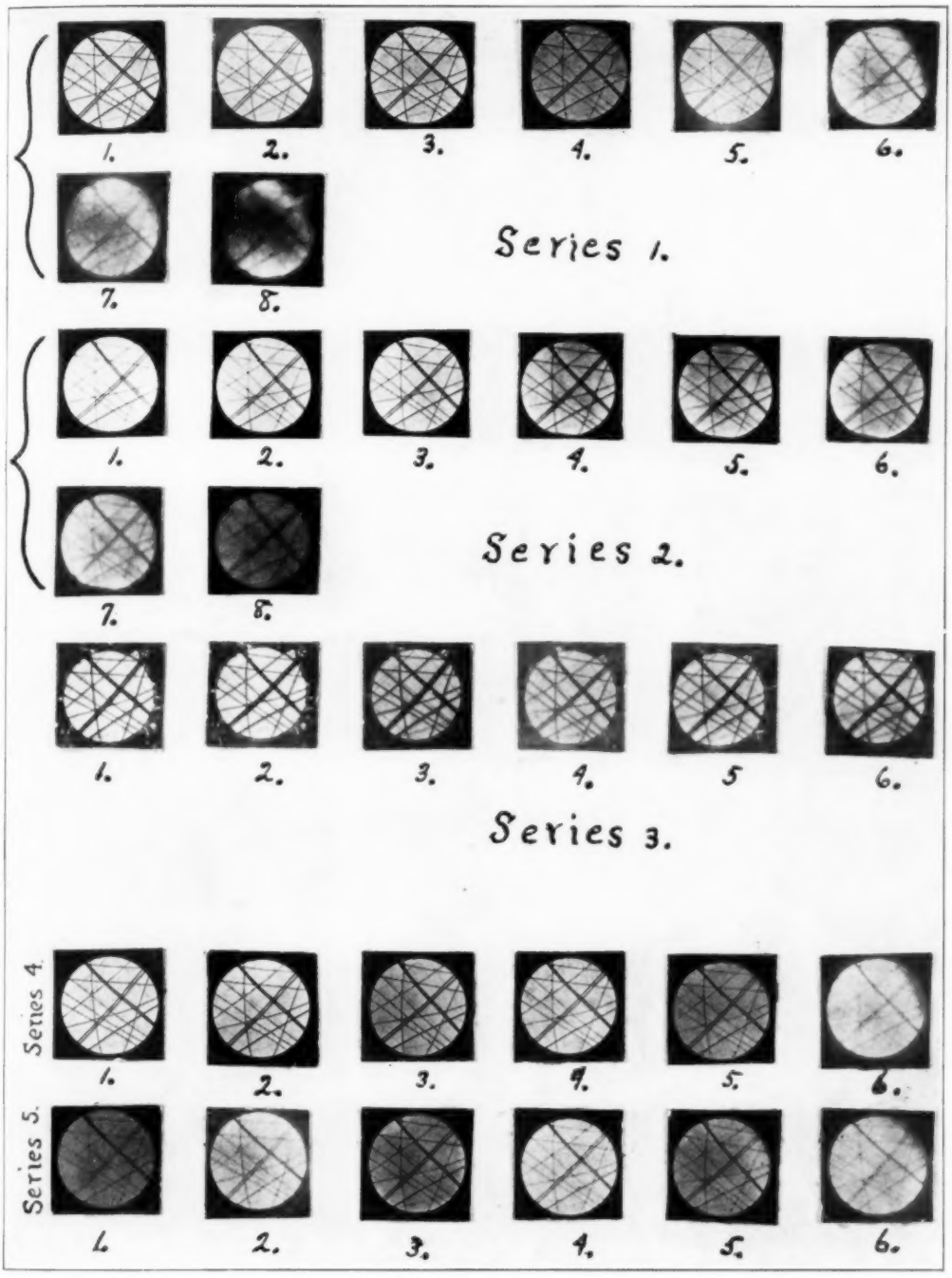

1.
2.
3.
4.
5.
6.
7.
8.
Series 1.
1.
2.
3.
4.
5.
6.
7.
8.
Series 2.
1.
2.
3.
4.
5
6.
Series 3.
Series 4.
1.
2.
3.
4.
5.
6.
Series 5.
1.
2.
3.
4.
5.
6.

PLATE XII

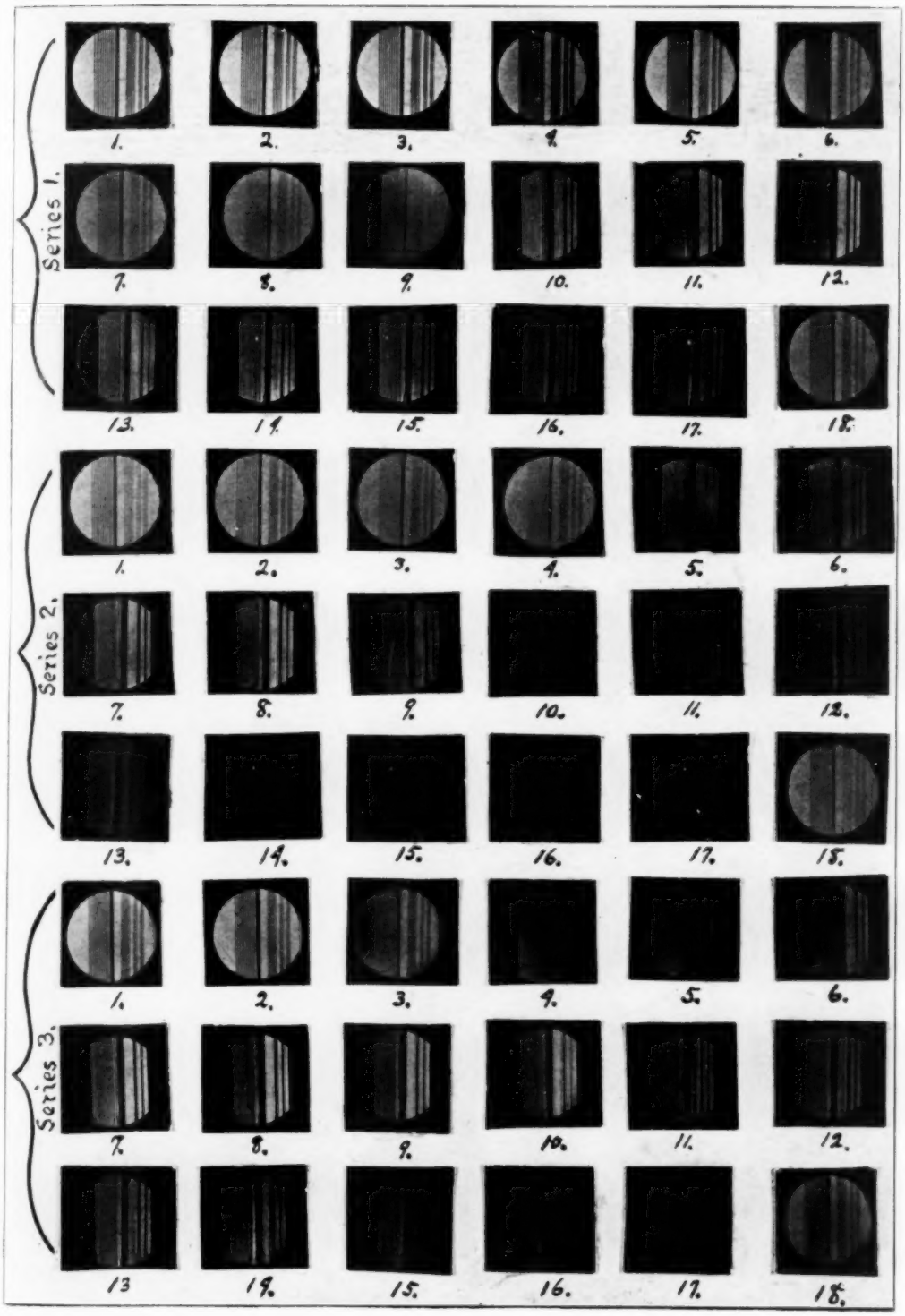
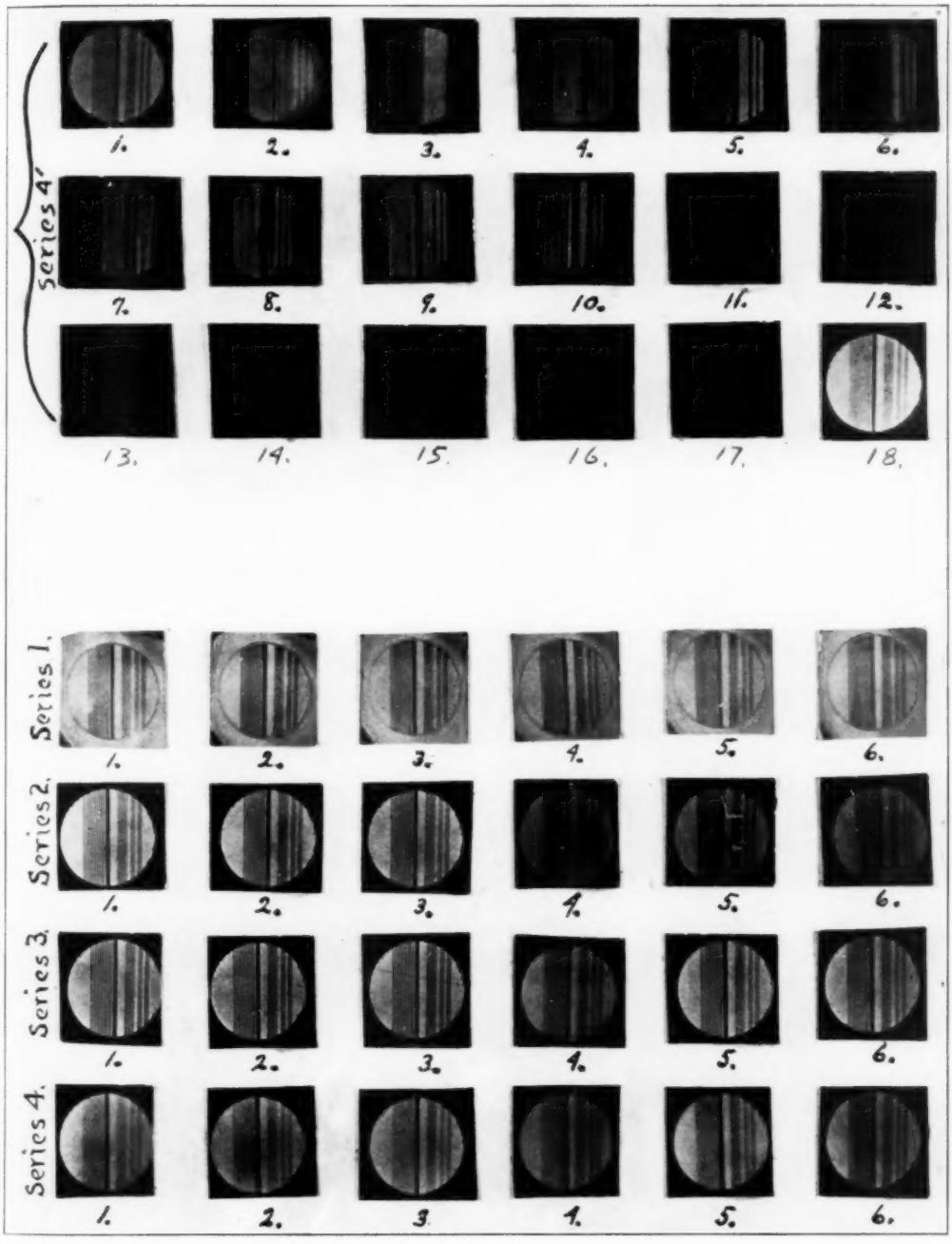

Series 1.
1. 2. 3. 4. 5. 6.
7. 8. 9. 10. 11. 12.
13. 14. 15. 16. 17. 18.
Series 2.
1. 2. 3. 4. 5. 6.
7. 8. 9. 10. 11. 12.
13. 14. 15. 16. 17. 18.
Series 3.
1. 2. 3. 4. 5. 6.
7. 8. 9. 10. 11. 12.
13 14. 15. 16. 17. 18.

PLATE XIII


Series 4'
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
Series 1.
1.
2.
3.
4.
5.
6.
Series 2.
1.
2.
3.
4.
5.
6.
Series 3.
1.
2.
3.
4.
5.
6.
Series 4.
1.
2.
3.
4.
5.
6.

because the illumination could not be made intense enough to affect the photographic plate within a reasonable time.

The points to be noticed in comparing number with like number in these three series are as follows: (1) With the exception of the cases of the very small apertures, the optimum images are better than the diffused light images. (2) There is very little difference between series 2 and series 3. The lines in series 3 are possibly a little broader, owing to the effect of shadows. This point will be discussed again in connection with Plate XIII. The similarity of series 2 and series 3 shows that we were justified in using Stoney's assumptions in the substitution of object No. 2 for object No. 1.

Realizing that I was dealing with ideal atmospheric conditions I arranged for disturbing the atmosphere while an exposure was being made. To accomplish this, two bunsen burners were placed between the object *O* and the camera *C*. I do not claim that this gave real atmospheric conditions, but it is quite evident that there would be some similarity between movements of the air in the laboratory and movements of the atmosphere when a telescope is being used in actual practice. Series 4 and 5, Plate XI, show the results obtained. Series 4 is made up of optimum images taken with the apertures used above, while series 5 was taken under the same conditions as series 4, except that the burners were lighted and the air was in motion. It is quite evident that motions or disturbances in the atmosphere affect the quality of the image. The final conclusions from these results will be left for the latter part of this paper.

Realizing that the results obtained by use of the complex object might be questioned, I constructed a more simple object and photographed it under the same conditions as the first. The second object used was a simple wire grating made by winding wire around two screws which were fastened in a frame. The screws were made with fifty threads to the inch. Wire of two sizes (0.15 mm and 0.37 mm) was wound upon these screws and soldered. The wire on one side was then cut away, and that which was left was tightened by turning two nuts in the frame. This grating was then placed in the position of the original object. The advantage in using the grating with vertical wires came from

the fact that its diffraction pattern consisted of a row of horizontal spectra, rather than confused and promiscuously scattered spectra, as was the condition in the case of the first object. As the camera aperture is moved to one side, nearly all of the diffraction pattern is admitted (a part at a time) into the camera. The composite photograph would be a truer representation of the actual image in this instance, for more of the pattern is used than in the series already discussed.

Plate XII contains three series, showing the different partial images obtained with apertures of varying diameters, the first one in each series representing the optimum image. Series 1 was taken with a 4-mm aperture, representing a 48-inch telescope; series 2 with a 2-mm aperture, representing a 24-inch telescope; series 3 with a 1-mm aperture, representing a 12-inch telescope; and series 4^1, Plate XIII, with a 0.6-mm aperture, representing a 7.2-inch telescope. It will be noticed in series 1 that as the aperture was moved sidewise across the field, one half-millimeter at a step, six good images were obtained, while series 2 contains four, series 3, two, and series 4^1, Plate XIII, only one. It would be expected, then, that the composite image would be much better in series 1 than in the other series, and this is what is observed when an examination is made of No. 18 in each series. In series 1, No. 18 is practically as distinct as the optimum No. 1. In series 4^1, Plate XIII, even the optimum No. 1 is indistinct. This would indicate that the aperture did not admit enough of the diffraction pattern to give a good image.

Plate XIII, lower half, shows the same effects as were shown by Plate XI. These photographs were taken with the following apertures: 4 mm, 3 mm, 2 mm, 1.25 mm, 1 mm, and 0.6 mm, representing respectively, the same sized telescopes as were mentioned in the previous tables. Series 1 was taken with reflected light, series 2 with transmitted diffused light, series 3 with light from a point-source which gives us optimum images, and series 4 was taken under the same conditions as in series 3, except that the atmosphere between the camera and the object was disturbed by means of two bunsen burners. There seems to be very little, if any, difference between series 1 and series 2. This again shows

that it makes no difference where the illumination is located and is a further vindication of Stoney's theory. The optimum images in series 3 are better than the corresponding images in series 1 and series 2. This is possibly more evident in Plate XIII than in Plate XI. Series 4 shows conclusively that movements in the atmosphere affect the quality of the image to a marked degree.

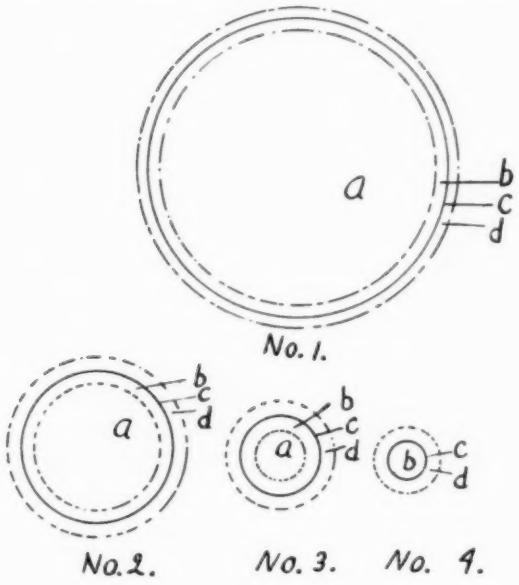


FIG. 2

In order to add clearness to the subject of partial images and further to explain the conclusions drawn from the photographs, Fig. 2 is inserted. The full line in these drawings indicates the relative size of the different apertures used. No. 1 represents a 48-inch, No. 2 a 24-inch, No. 3 a 12-inch, and No. 4 a 6-inch telescope.

The radii of the dotted circle are so chosen that the difference between each of these radii and the radius of the full circle would be equal to the relative radius of the fifth bright ring from the diffraction pattern of Mars. We can consider, then, that these three circles divide the space in and around the telescope objective

into three parts or divisions, *a*, *b*, and *d*. We then see that if the central bright spot of any of the infinite number of diffraction patterns from Mars falls within the space *a*, a good image will be formed. If the central bright spot falls within the space *b*, an image will be formed which will not be quite as good as those found in class *a*. As the central bright spot of the diffraction pattern passes over the edge of the telescope another class of images will be formed which will be much poorer than those of either class *a* or class *b*. This group of images is called class *c*. This corresponds to what has already been said in the previous paragraphs. If the central bright spot is not admitted into the camera, a very poor partial image is formed by the parts of the pattern that are admitted. As the central bright spot falls within the space *d*, only the outer diffraction rings enter the telescope and the images formed by them are poorer than those in either of the other classes. If the central bright spot falls outside of the space *d*, the essential parts of the pattern do not enter the telescope, and so we are not interested in them.

As has already been shown, the image formed in a telescope is the result of the superposition of the images from these four classes. If class *a* predominates the resultant image should be a comparatively good one, while if the poorer or inferior classes predominate, the resultant image will be of an inferior quality and thus will be indistinct.

By finding the areas of the different sections in each of the drawings of Fig. 2 some interesting comparisons can be made. Table III shows the relative values.

Now if we remember that the images formed by class *b* are fairly good partials and that those formed by class *a* are optimum partials, we can make use of Table III. When a 48-inch telescope is used, 75 per cent of the partial images making up the actual image are found in class *a*, while 86 per cent of the total are included in classes *a* and *b*. This would indicate that a 48-inch telescope should give a very good image of the disk of the planet Mars. These calculations are, of course, based on the image of the disk of Mars. The excellence of the image of the smaller details of the planet will be dependent on the relative size of these details as

compared with the size of the disk of the planet. No. 2 shows that when a 24-inch telescope is used, 56 per cent of the partials that make up the image are included in class *a* and 75 per cent in classes *a* and *b*. When we remember that the light from class *a* and class *b* is more intense than that from the other classes, we might

TABLE III

No.	Area sec. *a*	Area sec. *b*	Area sec. *d*	Area total	Area *a*+*b*
1	435	65	78	578	500
2	93	32	40	165	125
3	16	15	20	51	31
4	1.5	6.3	11	20.8	7.8

	Per cent *a* is of total area	Per cent *a*+*b* is of total area
1	75	86
2	56	75
3	31	60
4	7	37

expect a 24-inch telescope to give a reasonably good image. Nos. 3 and 4 indicate how far the quality of the image falls off when a 12-inch or a 6-inch telescope is used. In the latter case the aperture is so small that nearly all of class *a* is excluded. This is also in accord with experimental results, for it is found that when the aperture would not admit the whole of the bright part of the diffraction pattern, the image obtained was very indistinct.

CONCLUSIONS

1. The assumptions made at the beginning of this paper have been proved to be correct. This is first shown in Plates X and XI and later by Plates XII and XIII. There seems to be no difference between the photographs taken with transmitted diffused light and with reflected light. The sum of the partial images gives a composite image which seems to be of the same quality and shows the same details as the images obtained with diffused light. This point is of great importance because it shows that the resolution of the image into its different partials is permissible. It also shows that

the introduction of the lens in front of the object did not alter the optical conditions. If there is any difference between the composite image and the diffused light image, it may be accounted for by recalling that the composite image was taken with light of one wave-length, while white light was used when the diffused light images were obtained.

2. In order to obtain a good image the aperture of the telescope must admit all of the brighter parts of the diffraction pattern from the object. The essential part of the pattern would usually consist of the central bright spot and the first five or six bright rings. This conclusion is based on series 4, Plate XI, and series 3, Plate XIII. No. 6 in each of these series shows the effect of using an aperture that would not admit all of the brighter parts of the pattern.

3. Airy's formula can be used in making an estimate of the size of a telescope necessary to give a distinct image of a heavenly body. This is shown particularly in Fig. 2. The calculations and estimates made by use of this formula agree quite closely with the experimental results.

4. The quality of the image is not greatly improved by increasing the size of the telescope aperture indefinitely. Smaller details may be seen, however, with the larger telescopes. This is shown especially in series 3, Plate XIII. It will be observed that Nos. 1 and 2 are equally good. No. 1 was taken with a 4-mm aperture and No. 2 with a 3-mm aperture. No. 1, however, might have been able to show details that would not appear in No. 2, if smaller details had been present in the object.

5. Under ordinary conditions in nature, perfect images are not obtained. It has been shown that better images can be obtained with this experimental apparatus than can be obtained under normal conditions. In every case it was found that when an aperture large enough to admit the essential parts of the diffraction pattern was used the optimum images were better than those secured by the use of diffused light. This is shown in Plates XI and XIII.

6. If atmospheric conditions were perfect, details as small as 7 miles across, on the planet Mars, could be detected with a 24-inch telescope. This seems to be in agreement with the conclusions of Professor Lowell. He has detected certain details upon the sur-

face of the planet as readily with his 24-inch telescope as a number of other astronomers have seen with the larger instruments. This was due, not only to the undeniably good atmospheric conditions found in Arizona, but also, in accordance with the results of this paper, to the fact that his telescope is sufficiently large for these details. Whether these details represent canals or not is outside the scope of this paper.

7. Movements in the atmosphere affect the quality of the image to a marked degree. While real atmospheric conditions were not produced in the laboratory, yet the results obtained can be relied on to a certain extent.

PRACTICAL APPLICATIONS OF EXPERIMENTAL APPARATUS

1. The apparatus as set up in this experiment can be used in obtaining a photograph of any transparent object, such as lines on a glass plate. A better photograph can be obtained in this way than can be secured in the ordinary way with diffused light for illumination.

2. This apparatus can be used in studying the partial images from any transparent body and the contribution of each of these to the total image formed naturally with diffused light.

3. As advocated by Stoney and as shown in the results obtained in this paper, this apparatus can be used as a practical check on the work of astronomers, and, in fact, on any persons who view or photograph distant or minute objects.

In conclusion I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for their interest in the work, and, especially to Professor L. P. Sieg for suggesting the problem.

PHYSICAL LABORATORY
STATE UNIVERSITY OF IOWA
June 1914

A PHOTOGRAPHIC PERIODOGRAM OF THE SUN-SPOT NUMBERS

By A. E. DOUGLASS

In 1898 Professor Arthur Schuster suggested the construction of a periodogram which should display rhythmic time intervals in any series of measures, in the same manner that a spectrogram shows the space vibrations or waves in a beam of light. He analyzed the sun-spot numbers as an example, and showed how several periods, 4.38, 4.80, 8.36, 11.125, and 13.5 years, seemed to be operating,[1] some more intensely than others.

In 1913 Kimura[2] performed an analysis of the same by a method of successive approximations and derived the amplitudes of a large number of different periods, which when combined produce a result remarkably like the sun-spot curve since 1750. Some of his chief periods are: 82.2, 54.2, 20.03, 12.05, 11.114, 10.48, 9.99, and 8.55 years. Last year also Professor Turner took up the problem with consummate skill and untiring energy.[3] On thorough mathematical analysis in the first three papers, he finds that the main features of the sun-spot record can be represented by four periodic terms of approximately 8.3, 10.2, 11.4, and 14.7 years, but that their coefficients do not remain constant, and only the 11.4-year period is sensible at the present time. Meanwhile A. A. Michelson[4] applied the harmonic analyzer and determined the amplitudes of a considerable number of periods, such as 8.6,

[1] "On the Investigation of Hidden Periodicities, with Application to a Supposed 26-Day Period of Meteorological Phenomena," *Terrestrial Magnetism*, **3**, 13–41, 1898; "On the Periodogram of Magnetic Declination at Greenwich," *Cambridge Philosophical Society Transactions*, **18**, 107–135; "The Periodogram and Its Optical Analogy," *Proceedings of the Royal Society*, **77** A, 136–140, 1906; "On the Periodicities of Sun-Spots," *Philosophical Transactions of the Royal Society of London*, **206** A, 69–100, 1906.

[2] "On the Harmonic Analysis of Sunspot Relative Numbers," *Monthly Notices, R.A.S.*, **73**, 543, 1913.

[3] *Monthly Notices, R.A.S.*, **73**, 549, May 1913 (discussion of Kimura's paper); **73**, 714, sup. number, 1913 (applying the Fourier sequence); **74**, 16, November 1913 (continuing the last); **74**, 82, December 1913 (discussing discontinuities and the meteoric hypothesis).

[4] "Determination of Periodicities by the Harmonic Analyzer with an Application to the Sun-Spot Cycle," *Astrophysical Journal*, **38**, 268, 1913.

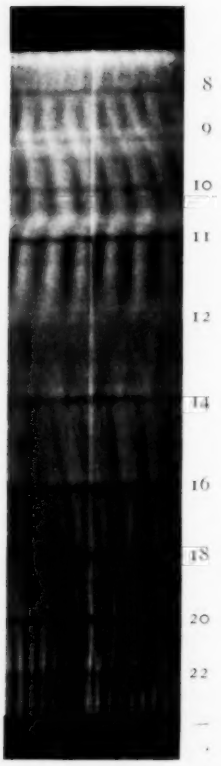


FIG. 1.—Periodogram of the sun-spot numbers. Corrugations show periods. The numbers give time of vibrations in years. The white line is the year 1830 and shows phase.

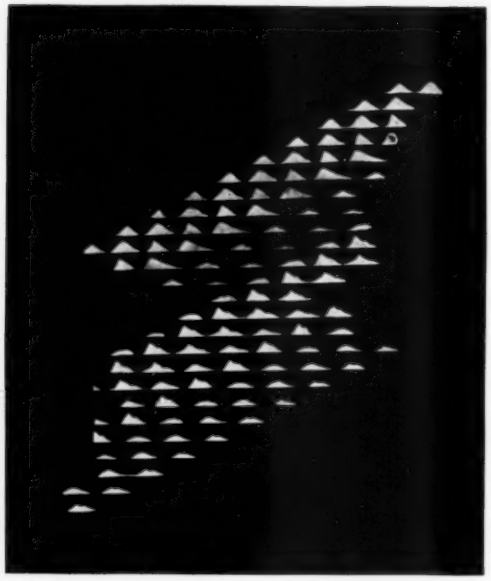


FIG. 2.—Diagram used in making the periodogram, consisting of the sun-spot curve mounted in multiple.

FIG. 3.—Figure 2 photographed out of focus to show discontinuities in the vertical lines.

10.3, 11.4, 15.1, 58, and 105 years, but most of these appear to him doubtful. He concludes, "Indeed it would seem that with the exception of the 11-year period and possibly a very long period (of the order of 100 years), the many periods found by previous investigators are illusory."

But Turner in his last article (December 1913) presents the matter from a new viewpoint. He investigates discontinuities in the series and finds that the breaks in the sun-spot series "are near the dates 1766, 1796, 1838, 1868, 1895, which are sufficiently close to those of Leonid periphelia to suggest a *vera causa*," and he discloses his hypothesis of the meteoric origin of sun-spot periodicity, a hypothesis of the greatest interest but not pertinent to the present subject.

The work described below adds little to the information obtained in the investigations alluded to, but it presents the sun-spot history in a new way and suggests perhaps a rapid means of carrying on preliminary studies of periodicity in any series of continuous records.

In Plate XIV, Fig. 1 is a periodogram of the sun-spot curve from 1755 to 1911, made by a photographic process in which the camera has done the additions for all the periods named beside it. The existence of a rhythm in any specified period is indicated by a beaded or corrugated effect. The corrugations are in fact the rhythmic vibrations of the curve. On a moment's examination this periodogram shows much of the information referred to above. The 11-year period is the most pronounced, yet not so superior to all others as would be expected. It may be of any duration from 11 to 11.8 years; 11.4 is a good average. There is obviously a period somewhere between 9.5 and 10.5 years and one between 8.0 and 8.8, but it is less conspicuous. Faint indications of periods are found near 14 years. The double of 8.4 is seen between 16 and 17 years. The double of the 10-year period shows near the 20, and at 22 the double of the 11 begins. Other photographs made at the same time[1] show the same periods extremely well, but were less satisfactory in some of the mechanical or photographic details.

[1] May 1913. H. H. Clayton had produced a periodogram by mechanical means some years previously. The writer here wishes to acknowledge his indebtedness to Professor E. C. Pickering for permission to use some of the facilities at Harvard College Observatory in this work.

In all cases the phase is located by the central white line which represents very closely the year 1830. In this photo-periodogram therefore, we have at once pictured to the eye some of the general results of the mathematical work. The method has the disadvantage that we cannot get absolute amplitudes, but perhaps some photometric work could give approximate values if it were worth trying. But relative amplitudes show at once; and we can judge, not only of the relative importance of different periods, but of the precision, or its lack, with which any period can be stated.

Fig. 2 shows a part of the process. It is the diagram from which the periodogram was photographed. In order to produce it the sun-spot curve was cut out in white paper and pasted in multiple, as is seen, on a black background. The left end of each of the upper 10 lines is the date 1755. Each successive line is moved 10 years to the left, so that passing from above vertically downward each successive line represents a date 10 years later than its predecessor. This continues until the whole period from 1755 to 1911 is covered; and the lower 10 lines show the latter date at their right ends. It is not necessary that any of the lines should be full length, as we use only a part of each. Now, by passing the eye downward from the top, a period near 10 years will show itself at once by the successive crests in vertical alignment. If the crests form a line at some angle to the vertical, then the period they indicate is not exactly 10 years; it is more than 10 if the slant is to the right and less than 10 if the slant is to the left. The horizontal lines are spaced the equivalent of 5 years, hence if we measure the angle A made by a vertical line and a line joining two crests in successive horizontal lines, we find the period indicated is

$$P = 10 \text{ years} + 5 \tan A$$

where A is positive if measured on a slant to the right.

Since each angle with the vertical represents a different period, it was only necessary to mount this diagram on an axis with clock work and slowly rotate it, in front of a camera with a cylindrical lens over its objective and a narrow horizontal slit in the focal plane and a sensitive plate passing slowly downward across the slit, to produce the periodogram. The cylindrical lens with

horizontal axis summates the duplicate curves in whatever line happens to be vertical. Of course there is a practical limit to the different angles at which the diagram may be viewed. An angle too far in one direction, making the tested period very small, would require a great number of duplications of the curve, while too great an angle the other way, making the tested period very large, catches the curve used here in a non-symmetrical form and introduces error. In the periodograms actually made of the sun-spot curve the minimum period tested was 7 years and the maximum 24 years. One notes especially that this is a continuous process and that all periods from the minimum to the maximum are tested.

In the arrangement described above, there are several limitations to the accuracy of results. First, the curves are non-symmetrical about their horizontal bases, and, when summated in a slant line far from the vertical, crests are thrown to one side of their proper place. Plotting the curves above and below a line would improve the result. Secondly, the resolving power decreases as the slant either way increases: the plot must be on larger and larger scale to overcome this. Thirdly, the wider the slit in the focal plane, the less the resolving power. Fourthly, there is a photographic limitation due to the failure of the photograph to show slight contrasts, and, fifthly, the eye cannot ordinarily detect contrasts under about 10 per cent.

When one applies this method as here described to other curves, a new condition presents itself, for the percentage variation in most curves is far less than in the sun-spot numbers. One must therefore cut off the lower part of the plot between the zero line and the lower extremes of the curve; for example, it would be thus in a series of barometer readings. However, in spite of these limitations the photographic periodogram does give important information quickly and plainly to the eye and at little cost, and thus can serve as a guide, showing when more refined methods are desirable.

But in common with any single mathematical treatment, the periodogram has one defect, it assumes the variables to be continuous throughout the series. It seems to the writer that Turner

has caught an important secret of the sun-spot problem, namely, that the variables are discontinuous. One can see from his papers how great a labor it has taken to reach that conclusion. This accounts for much of the discordance between investigators and the disappointment one has felt in the lack of definite result and a basis of prediction for the future.

Now it would take several periodograms of the type here presented to show this discontinuity, but the method here given, with a slight change, does show the whole history of the sun-spot discontinuities at a glance. Fig. 3 is a photograph of Fig. 2, taken out of focus for the purpose of calling attention to certain general features. In Fig. 2 the eye naturally turns to the sharp outlines and notes its minute details. In Fig. 3 the crests of 2 are changed into large blotches connecting somewhat with their nearest neighbors and varying in intensity. The sun-spot sequence appears in each nearly vertical line of blotches. Having a number of exactly similar lines side by side, the irregularities are repeated in each and thus strike the consciousness with the effect of repeated blows. These irregularities are the discontinuities referred to by Turner in connection with his hypothesis. It is evident at a glance that the sun-spot sequence divides itself into three parts, namely, a 9.3-year period, 1750–1790, then an interval of readjustment, 1800–1830, with a 13-year period, and lastly an 11.1-year period lasting to the present time (values approximate).[1] But the latter is not perfectly constant, for after 1870 there is a change in intensity. The breaks thus shown and Turner's dates of discontinuity are here compared.

Periodogram	Turner
	1766
Between 1788 and 1804	1796
" 1830 and 1837	1838
" 1870 and 1884	1868
	1895

[1] In discussing the periodicities of sun-spots (pp. 75–78) Schuster divided his 150 years, from 1750 to 1900, into two nearly equal parts. He found in the first part two periods of $9\frac{1}{4}$ and $13\frac{3}{4}$ years acting, successively, and in the second part, a period of 11.1 years. This is shown graphically in Fig. 3.

By means of this diagram, one can discover at a glance the origin of many of the periods which Michelson thought were illusory, and in which he was largely right. We can plainly see a 9.3-year period in the early part of the curve; let us call this part of the sequence A_n; and let us call its broken continuation near the center B_n, and the lower and later part giving the 11.1-year period, C_n. Thus we get at once three periods 9.3, 11.1, and something over 13 years. If now we bring the average A_n into line with the average C_n as the periodogram does we get 11.4 years. If we bring the average A_n into line with the average C_{n-1} we get close to 10 years. If we bring into line A_n and the heavier parts of C_{n-2} we get 8.4 years or thereabouts. And at 5.6 years we find a period which is just half of C_n and at 4.7 we find the half of A_n, and so on. It is like a checker board of trees in an orchard; they line up in a number of directions with more or less intensity. But the diagram in Fig. 3 helps remove some of the complexity of the sun-spot problem. It shows us that while these various periods are apparent, yet many are illusory as Michelson said. The diagram supplies a basis for profitable judgment in the matter. Hence to avoid just such awkward cases as the sun-spot curve, it is here presented as a necessary accompaniment of the periodogram in any doubtful cases.

UNIVERSITY OF ARIZONA
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WAVE-LENGTHS OF THE CHIEF LINES OF NITROGEN AND OXYGEN IN THE REGION λ 3880 TO λ 4700

By JOHN S. CLARK

In a note to the Astronomical and Astrophysical Society of America in August 1911, Sebastian Albrecht drew attention to the urgent need of additional investigation in the laboratory of the wave-lengths of the lines due to nitrogen, oxygen, and silicon, in order to obtain sufficiently accurate data for his work on stellar radial velocities. It was pointed out that the two best laboratory determinations differed from each other by many times the probable errors of the wave-lengths of these lines in stellar spectra relative to the standard lines of carbon (λ 4267.280), hydrogen (λ 4340.634), helium (λ 4471.693), and magnesium (λ 4481.375), and that this uncertainty as to the wave-lengths caused the radial velocities of the B-type stars to be uncertain by possibly as much as 4 or 5 km per second. The present investigation was carried out with a view to determining accurately the wave-lengths of the chief lines of nitrogen and oxygen in the region usually studied in stellar spectrograms, viz., λ 3880 to λ 4700.

It was found that suitably sharp nitrogen and oxygen lines were obtained when a highly condensed discharge was passed through dry air at rather low pressure in a capillary vacuum tube, and this method was used as a source of the lines. The spectra were photographed by means of a 10-foot concave grating, mounted according to the method described by Eagle,[1] and the third-order spectrum, giving a mean dispersion of 1.7 A per mm, was used throughout. The internal shutter of the instrument permitted two comparison spectra to be partially superimposed on the spectrum to be measured, and so the iron arc was exposed twice, once before and once after the long exposure. Thus it was arranged that any displacement, due to change of temperature or other cause, occurring during the exposure, might be detected. The temperature of the air near the grating was measured by means of a resistance thermometer and recorded on the chart of a Callendar recorder. Now the

[1] *Astrophysical Journal*, 31, 120, 1910.

coefficient of linear expansion of speculum metal is 0.0000193 per degree C. Hence at λ 4100, the middle of the region examined on one plate, a temperature change of 0°.17 C. (which was the total observed rise in temperature during the particular four hours' exposure) would correspond with a displacement of 0.0134 A. The observed displacement, as deduced from the measurements with reference to the two iron comparisons, was in the correct direction, but its average value was only 0.005 A, and as the greater part of the rise in temperature occurred at the beginning of the exposure (as shown by the temperature chart) the wave-lengths obtained in this particular case from the second iron comparison were given greater weight (in the proportion of 3 to 2) in estimating the final wave-lengths.

In measuring the negatives the lines were divided into groups extending over not more than 15 A, and these groups were measured with red to the right and left in succession, first with one iron comparison, and then with the other, two independent values for the wave-length thus being obtained from each plate. The standard iron lines used were in every case "A lines" given by Burns,[1] and the measured wave-lengths were corrected where necessary by means of parabolic curves of errors obtained from the measured iron lines.

The results are given in the appended tables, and the previous measures of Neovius[2] and of Frost and Adams[3] are included for comparison. The latter are given in terms of Rowland's units, and to facilitate comparison the present measures have also been converted from the International to the Rowland scale by means of a curve taken from Kayser's *Handbuch der Spectroscopie* (**6**, 890).

Column 4 in the tables gives the probable errors derived from successive measures of the lines. The probable errors due to faulty setting of the cross-wires were worked out from the actual settings by means of the usual formula, viz.,

$$\text{Probable error of mean} = 0.6745 \sqrt{\frac{\Sigma(d^2)}{n(n-1)}}$$

[1] *Zeitsch. für wiss. Phot.*, **12**, 6, 1913, and *Lick Observatory Bulletin*, **8**, 247, 1913.

[2] *Bihang. till K. Svenska Vet.-Akad. Handl.*, Bd. **17**, No. 8, 56, 1891.

[3] *Astrophysical Journal*, **16**, 120, 1902.

TABLE I

NITROGEN

CLARK				PREVIOUS MEASURES			REMARKS
λ (International Units) 1	λ (Rowland) 2	Intensity (Max. 10) 3	Probable Errors 4	Neovius 5	Intensity 6	Frost and Adams 7	8
3914.138.....	3914.28	< 1	±.004	.....	.....	.....	Not tabulated by Neovius
14.301.....	14.45	2	.001	.....	.....	.....	
19.003.....	19.15	3	.001	3919.2	10	.....	
55.851.....	56.00	4	.004	56.1	7	.....	
94.995.....	95.15	10	.005	95.2	12	.....	Broad line
4026.087.....	4026.23	1	.004	4025.9	2n	.....	
35.090.....	35.24	3	.001	35.2	5n	.....	
41.325.....	41.47	6	.01	41.4	6n	.....	
97.327.....	97.48	10	.004	97.4	8n	.....	Lockyer's enhanced lines (4097.45, 4103.5)
4103.393.....	4103.54	6	.004	4103.4	4n	.....	
19.222.....	19.37	4	.003	19.3	9	.....	Also oxygen line according to Neovius
.....	24	< 1	.....	24.0	5	.....	Not measured
33.654.....	33.80	1	.005	34.2	6	.....	
45.759.....	45.91	2	.001	45.8	7	.....	
.....	52	< 1	.....	52.0	5	.....	Not measured
76.163.....	76.31	2	.001	76.7	5n	.....	
4227.738.....	4227.90	1	.001	4228.5	6n	.....	
36.99.....	37.15	1n	.01	37.0	6n	.....	Close diffuse doublet
41.787.....	41.94	8	.003	42.0	6n	.....	
4348.078.....	4348.24	1	.003	4347.9	6	4348.13	
.....	.....	.....	.....	4426.1	5	.....	
4447.035.....	4447.20	10	.003	47.2	12	4447.16	
.....	.....	.....	.....	60.0	3	.....	
4510.91.....	4511.08	1	.01	.....	.....	.....	
14.865.....	15.04	2	.008	4514.8	3	4515.08	
30.421.....	30.59	2	.005	30.3	6n	.....	
4601.490.....	4601.66	4	.002	4601.3	9	4601.63	
07.167.....	07.34	3	.004	07.2	8	07.31	
13.884.....	14.06	2	.002	14.2	8	14.03	
21.405.....	21.58	3	.002	22.0	9	21.55	
30.551.....	30.73	8	.003	30.9	12	30.70	
34.165.....	34.34	1	.01	34.0	3	.....	Diffuse line (possibly poor measure)
40.649.....	40.82	3	.005	40.5	1	.....	
43.106.....	43.28	4	.002	43.4	9	43.24	
50.853.....	51.03	1	.01	51.0	4	50.93	Also given as an oxygen line

TABLE II

OXYGEN

CLARK				PREVIOUS MEASURES			REMARKS
λ (International Units) 1	λ (Rowland) 2	Intensity (Max. 10) 3	Probable Errors 4	Lunt 5	Intensity (Max. 10) 6	Frost and Adams 7	8
3882.194	3882.34	1	±.004	3882.37	6	.....	
3911.951	3912.10	2	.001	3912.19	7	.....	
45.033	45.18	< 1	.007	45.26	6	.....	
54.368	54.51	3	.003	54.64	7	.....	
73.266	73.41	5	.001	73.51	9	.....	
82.725	82.87	1	.007	82.93	6	.....	
4069.635	4069.78	3	.01	.....	.....	.....	Not given by Lunt or Neovius, but probably due to oxygen
69.903	70.05	4	.005	4070.07	10	.....	
72.156	72.31	6	.002	72.45	10	.....	
75.869	76.02	8	.001	76.09	10	.....	
4105.001	4105.15	1	.004	4105.15	6	.....	
19.222	19.37	4	.003	19.51	8	.....	Also given as nitrogen
20.267	20.42	1	.01	20.61	6	.....	
32.99	33.14	1	.01	33.06	5	.....	Diffuse line—possibly poor measure
53.310	53.46	1	.001	53.64	6	.....	
85.453	85.61	3	.001	85.72	6	.....	
89.793	89.95	3	.003	90.06	7	.....	
4317.160	4317.32	2	.01	4317.25	6	4317.27	
19.647	19.81	2	.005	19.79	6	19.76	
45.570	45.73	2	.006	45.75	6	45.68	
47.429	47.59	2	.008	47.54	6	47.62	
49.435	49.60	4	.007	49.52	8	49.54	
51.275	51.44	3	.004	51.35	7	51.50	
66.906	67.07	2	.01	67.03	6	67.01	
4414.888	4415.05	5	.01	4415.08	10	4415.08	
16.974	17.14	4	.005	17.02	9	17.12	
4590.983	4591.16	3	.01	4590.91	8	4591.07	
96.189	96.36	2	.005	96.05	7	96.29	
4638.865	4639.04	1	.005	4638.88	5	4638.94	
41.827	42.00	3	.005	41.84	8	41.89	
49.148	49.32	6	.001	49.06	9	49.25	
50.853	51.03	1	.01	50.95	5	50.93	Also given as a nitrogen line
61.650	61.83	2	.002	61.64	5	61.73	
76.246	76.42	1	.008	76.22	5	.....	

but these errors were always less than 0.001, and in many cases less than 0.0005 for the better lines. The figures tabulated are estimates of the probable errors derived from successive measurements of the wave-length, due consideration being paid to the character of the line and the difficulty of measuring it accurately. As a matter of fact, however, the lines, being produced under the conditions of low pressure and high-tension discharge, were in general exceedingly sharp and thus well adapted to accurate measurements.

The foregoing measurements were carried out in the spectroscopic laboratory of the Imperial College of Science and Technology, London, under the direction of Professor Fowler, F.R.S., to whom the author is indebted for constant interest during the measurement, and advice in the preparation of the paper.

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